A New and Novel Approach for Understanding and Flying a Precision On-Pylon Turn

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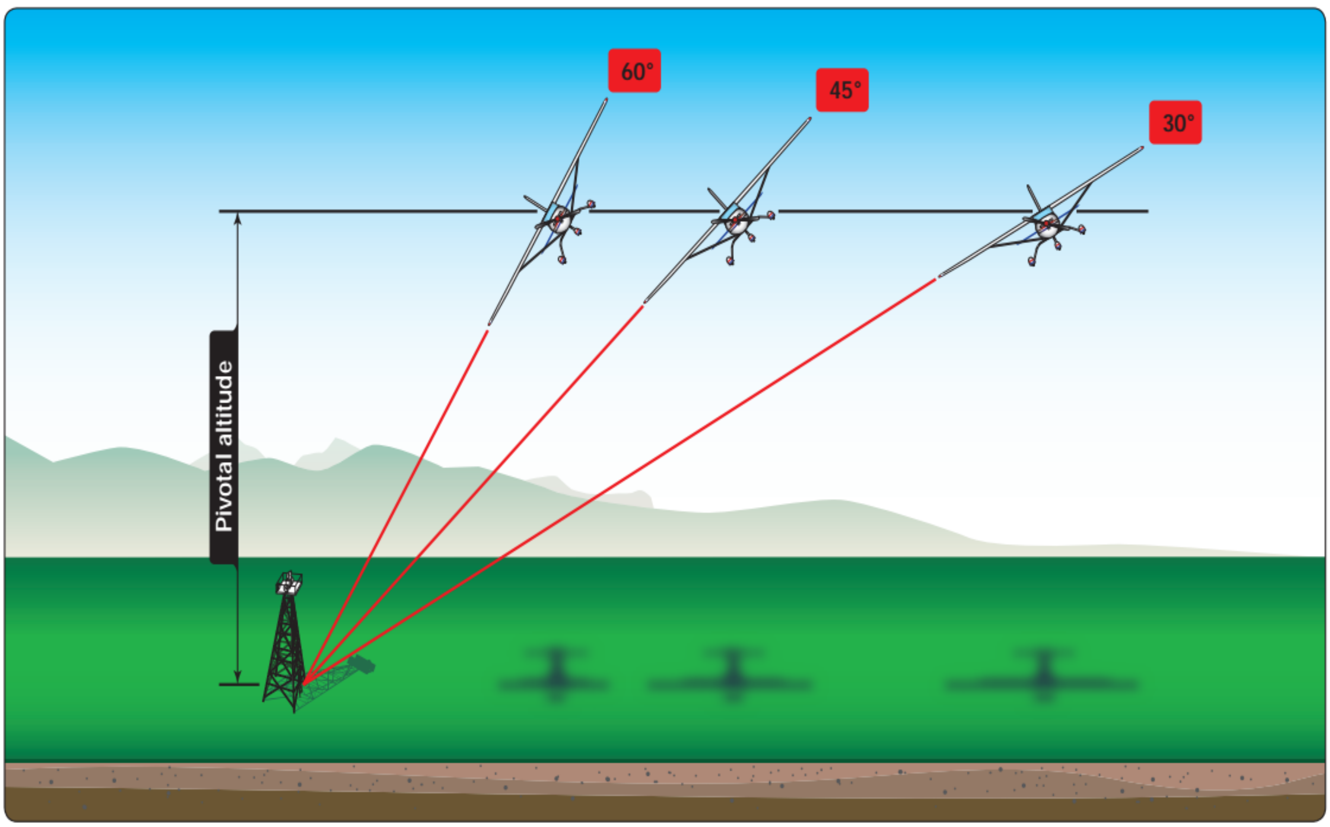

Summary

In this White Paper we derive the complete solution to the On-Pylon Turn maneuver. The equation for the pivotal altitude is derived from simple classical dynamics. The equation for the pivotal altitude given in the FAA Airplane Flying Handbook (8083-3A, 2004) is

$$h = \frac{V^2{}_{TAS}}{11.3}$$

Where V_{TAS} is the TAS in knots. However in the latest version (8083-3B, 2016), the pivotal altitude is given by

$$h = \frac{V_G{}^2}{11.3}$$

Where V_G is the groundspeed in knots. Both of the above equations for the pivotal altitude are shown to be in error when compared to the derived solution, which is given by

$$h = \frac{V_{TAS} V_\theta}{11.3}$$

Where V_θ is the groundspeed in knots in the transverse direction (i.e. it does not include the radial component of the wind). Here

$$V_\theta = V_{TAS}(1 + \bar{V}_W \, \text{Cos}\, \theta)$$

Where $\bar{V}_W$ is the windspeed ratio given by $\frac{V_{Wind}}{V_{TAS}}$, and θ is the angular position measured from the downwind position ($\theta = 0$). The complete solution of the On-Pylon Turn provides the following information as a function of θ:

(1) Required bank angle
(2) Required rate of turn
(3) Required rate of climb/descent
(4) Required distance from the pylon
(5) Required wind correction angle (WCA)

We show that the key parameter in (1)-(5) is the windspeed ratio. In addition, the solution of the On-Pylon Turn maneuver shows that the maximum bank angle occurs on the downwind position , where the aircraft is closest to the pylon, while the minimum bank angle occurs on the upwind position ($\theta = 180$), where the aircraft is farthest from

the pylon. The track of the aircraft while holding the pylon is shown to be an ellipse with the eccentricity of the ellipse determined by windspeed ratio $\bar{V}_W$.

The origin of the visual cues that the Pilot observes which indicate whether the aircraft is above or below the pivotal altitude are also derived based on physical arguments. The formulas that provide the results for items listed in (1)-(5) above are shown to provide a significant amount of information that allows the Pilot/Instructor to plan and execute the maneuver, such that the bank angle does not exceed a specified value on the downwind and the required maximum rate of climb is below the maximum rate of climb of the aircraft. We show that for large windspeed ratios (i.e. $\geq$ 0.2) the required rate of climb to hold the pylon can easily exceed the performance of a C-172 when flying the maneuver at 90 KTAS.

Finally, we show that the On-Pylon Turn maneuver can be flown accurately in the presence of a wind, using a constant pivotal altitude, if power is utilized to vary the TAS around the pylon. A comparison of the aircraft track, the bank angle, and turn rate, for various windspeed ratios is made for both the constant TAS and constant pivotal altitude methods.

1.0 Introduction

Ground reference maneuvers are maneuvers that are required for both the Private Pilot and the Commercial Pilot Certificate. In the case of the Private Pilot Certificate, the Pilot must satisfactorily demonstrate: (a) Rectangular Course, (b) S-Turns across a Road, and (c) Turns around a Point. In the case of the Commercial Pilot Certificate, the Pilot must satisfactorily demonstrate Eights-On Pylons. In a previous White Paper (Ref. 1), we discussed in significant detail, Turns around a Point. We showed the key parameter in all ground reference maneuvers is the windspeed ratio, which is defined as the ratio of the windspeed to the aircraft's TAS.

Prior to teaching Eights-on-Pylons, we usually introduce the Student to the simple On-Pylon Turn maneuver. This maneuver allows the Student to understand the concept of pivotal altitude, which is the altitude that allows an imaginary line extended from the Pilot's eye to the pylon, which is parallel to the lateral axis of the aircraft, appears to pivot around the pylon. It also introduces the Pilot to the key observations which are used to determine whether the aircraft is above or below the correct pivotal altitude. In the Airplane Flying Handbook (FAA-H-8083-3A, 2004, Ref. 2), the pivotal altitude was estimated as the square of the TAS divided by 11.3, where the TAS is in knots. However, in the latest version of the Airplane Flying Handbook (FAA-H-8083-3B, 2016, Ref. 3), the pivotal altitude was estimated as the square of the groundspeed divided by 11.3, where the groundspeed is in knots. Although the results are identical when the windspeed is zero, we will show that the pivotal altitude in the presence of a wind is incorrect as given in both versions of the Airplane Flying Handbook.

In Section 2 we introduce the dynamics of the turn and the concept of pivotal altitude. We derive the correct expression for the Centripetal acceleration during the On-Pylon Turn maneuver. This result is then used to derive the correct formula for the pivotal altitude. In Section 3 we derive the ground track of the On-Pylon Turn maneuver as a function of the angular position relative to the downwind. In Section 4 we derive equations for the required (1) bank angle, (2) rate of turn and (3) rate of climb/descent as a function of the position relative to the downwind in order for the Pilot to hold the pylon. We also show how to select the combination of both TAS and the radius on the downwind, such that the aircraft can meet both the maximum required rate of climb during the maneuver while keeping the bank angle below a specified maximum value. In Section 5 we describe how to determine whether the aircraft is above or below the pivotal altitude, and what corrections are necessary to bring the aircraft back on the pylon. In Section 6 we discuss a different method for holding the pylon in the presence of a wind, while flying at a constant pivotal altitude and varying the TAS. We summarize the results in Section 7 and provide references in Section 8.

In addition, we have highlighted important formulas and statements in red, which are the takeaways that all Pilots/Instructors should understand in order to fly the On-Pylon Turn maneuver with precision.

2.0 Dynamics of the On-Pylon Turn

In order to determine the formula for the pivotal altitude, it is necessary for one to determine the Centripetal acceleration that arises during the On-Pylon Turn. Recall that during the On-Pylon Turn, an imaginary line extended from the Pilot's eye to the pylon, which is parallel to the lateral axis of the aircraft, appears to pivot around the pylon. Under these conditions, it is beneficial to analyze the planar motion of the aircraft in a rotating radial-transverse coordinate system. Figure 1 shows the rotating radial-transverse coordinate system, where $\hat{\mathrm{i}}_r$ is the unit vector in the radial direction and $\hat{\mathbf{i}}_\theta$ is the unit vector in the transverse direction. The pylon is located at O and the aircraft is located is point **p,** and is moving along the track **s**. Here θ is the angular position around the pylon. Note that the radial unit vector and transverse unit vector are perpendicular to each other.

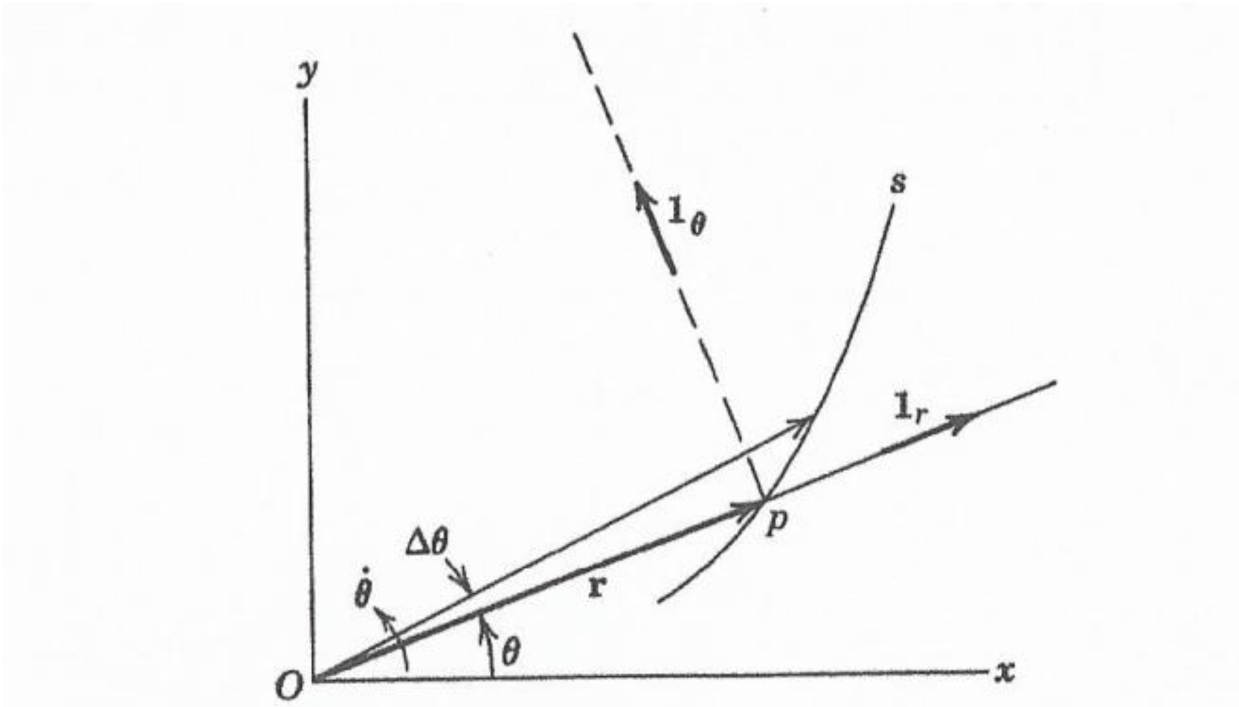

Figure 1: Radial-Transverse Coordinate System

Using this coordinate system, the rate of rotation of the aircraft around the pylon is just

$$\dot{\theta} = \frac{d\theta}{dt} = \frac{V_\theta}{r} \qquad (1)$$

Here, V_θ is the transverse velocity of the aircraft, and r is the radial distance from the pylon to the aircraft. In order to determine V_θ, it is necessary to add the TAS of the aircraft to the component of the wind in the transverse direction. Figure 2 shows the wind, V_W aligned in the positive-y direction, and the angle θ which is measured from the downwind position (θ =0) in a counter-clockwise direction. Here, $\dot{\theta}$ is the non-uniform rotational rate of the aircraft around the pylon. In addition, the unit vector $\hat{\mathbf{i}}_t$ is directed along the tangent to the curve **s** at the point **p**, which is the direction of the actual ground track of the aircraft.

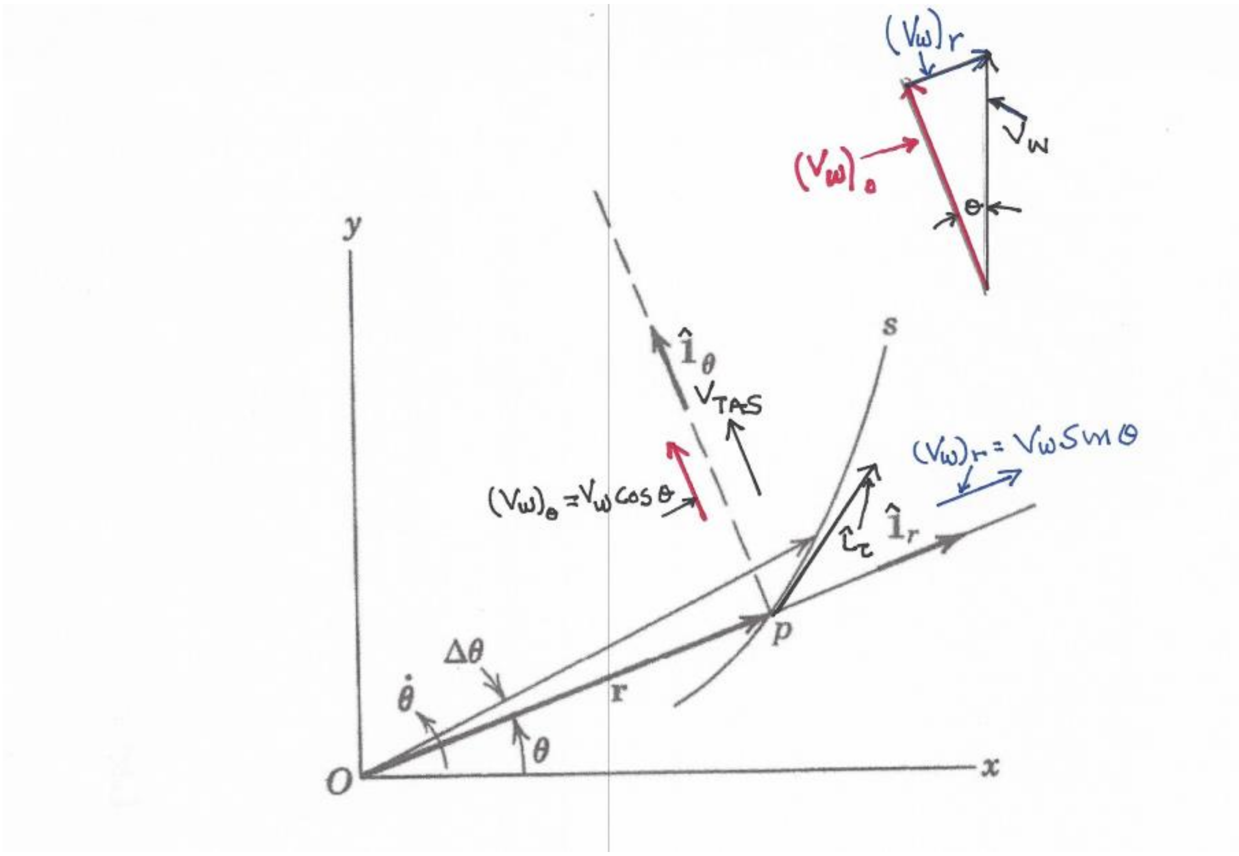

Figure 2: Radial-Transverse Coordinate Showing Components of Velocity

Figure 3 shows the components of the velocity of the aircraft in the $\hat{\mathbf{i}}_r$, $\hat{\mathbf{i}}_\theta$ and $\hat{\mathbf{i}}_t$ directions. Here V_G is the aircraft groundspeed and includes both the radial and transverse components of the velocity. Recall the rotational rate is only dependent on the transverse groundspeed.

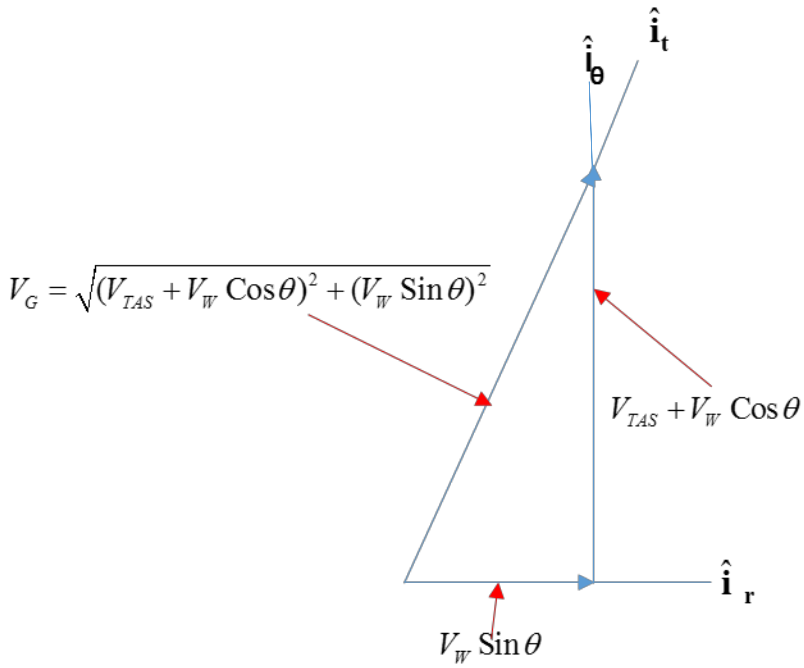

Figure 3: Components of the Ground Speed in the $\hat{\mathbf{i}}_r$, $\hat{\mathbf{i}}_\theta$, $\hat{\mathbf{i}}_t$ Coordinate System

We now need to express the acceleration of the aircraft in the rotating coordinate system. The acceleration of an aircraft in a rotating coordinate system is straight forward to derive and is shown in many classical textbooks on dynamics, such as Ref. 4. However, we will provide the derivation below in order for the reader to understand how one obtains the correct Centripetal acceleration.

If the aircraft is moving along a curve around the pylon, the position vector $\vec{\boldsymbol{r}}$ is given by

$$\vec{\boldsymbol{r}} = r\hat{i}_r \qquad (2)$$

Where r is the radial distance from the pylon to the center of gravity of the aircraft, and $\hat{\boldsymbol{i}}_r$ is the unit vector in the radial direction. The velocity of the aircraft is given by the derivative of the position vector with respect to time, i.e.

$$\frac{d\vec{\boldsymbol{r}}}{dt} = \dot{r}\hat{\boldsymbol{i}}_r + r\frac{d\hat{\boldsymbol{i}}_r}{dt} \qquad (3)$$

Note that second term in eq.(3) is the derivative of the unit vector $\hat{\mathbf{i}}_r$ with respect to time. One needs to include this term due to the fact that the coordinate system is rotating and the direction of $\hat{\mathbf{i}}_r$ is changing with time. Since the coordinate system is rotating about an axis perpendicular to the x-y plane, the derivative of $\hat{\mathbf{i}}_r$ is just the cross product of the two vectors $\dot{\boldsymbol{\theta}}$ and $\hat{\mathbf{i}}_r$. The orientation of this cross product is perpendicular to $\vec{\boldsymbol{r}}$ and is subject to the right-hand rule. Thus, we can express the derivative of $\hat{\mathbf{i}}_r$ with respect to time as

$$\frac{d\hat{\mathbf{i}}_r}{dt} = \dot{\boldsymbol{\theta}} \times \hat{\mathbf{i}}_r = \dot{\theta}\hat{\mathbf{i}}_\theta \qquad (4)$$

Here, $\hat{\mathbf{i}}_\theta$ is a unit vector in the direction perpendicular to $\vec{\boldsymbol{r}}$ .Thus eq. (3) becomes

$$\frac{d\vec{\boldsymbol{r}}}{dt} = \vec{\boldsymbol{v}} = \dot{r}\hat{\mathbf{i}}_r + r\dot{\theta}\hat{\mathbf{i}}_\theta \qquad (5)$$

Therefore, eq. (5) gives the velocity in terms of the radial and transverse components. The acceleration of the aircraft around the pylon is obtained by differentiating the velocity vector with respect to time, i.e.

$$\vec{\boldsymbol{a}} = \ddot{\vec{\boldsymbol{r}}} = \ddot{r}\hat{\mathbf{i}}_r + \dot{r}\frac{d\hat{\mathbf{i}}_r}{dt} + (r\ddot{\theta} + \dot{r}\dot{\theta})\hat{\mathbf{i}}_\theta + r\dot{\theta}\frac{d\hat{\mathbf{i}}_\theta}{dt} \qquad (6)$$

In differentiating eq. (5), we have introduced the term $\frac{d\hat{\mathbf{i}}_\theta}{dt}$, which can be expressed as

$$\frac{d\mathbf{i}_\theta}{dt} = \dot{\boldsymbol{\theta}} \times \hat{\mathbf{\imath}}_\theta = -\dot{\theta}\hat{\mathbf{i}}_r \qquad (7)$$

Note that the negative sign in eq.(7) is due to the sign of the cross product being determined by the right-hand rule. Equation (6) can be simplified to give the final result for the acceleration $\vec{\boldsymbol{a}}$, i.e.

$$\vec{\boldsymbol{a}} = (\ddot{r} - r\dot{\theta}^2)\hat{\mathbf{i}}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\hat{\mathbf{i}}_\theta \qquad (8)$$

Equation (8) provides the resultant acceleration in both the radial and transverse directions. The radial component is called the Centripetal acceleration, and the transverse component is the transverse acceleration. However, since the coordinate system is rotating with the non-uniform rotation rate, one would expect the transverse acceleration relative to the rotating coordinate system to be identically zero in the case of the aircraft flying at constant TAS. This fact will be borne out in the analysis below.

In Figure 2, the wind is oriented in the positive-y direction, such that, $\theta = 0$ corresponds to the downwind position, and $\theta = 180$ degrees corresponds to the upwind position. One can then express the radial and transverse components of the wind as follows:

$$(V_W)_r = V_W \sin\theta$$
$$(V_W)_\theta = V_W \operatorname{Cos}\theta \qquad (9)$$

The velocity in the transverse direction is given by

$$V_\theta = V_{TAS} + (V_W)_\theta = V_{TAS} + V_W \operatorname{Cos}\theta \qquad (10)$$

The velocity in the radial direction is given by

$$\dot{r} = V_r = V_W \sin\theta \qquad (11)$$

We can express the transverse acceleration component in eq. (8) as follows:

$$(r\ddot{\theta} + 2\dot{r}\dot{\theta}) = \{\frac{d(r\dot{\theta})}{dt} - \dot{r}\dot{\theta}\} + 2\dot{r}\dot{\theta} = \frac{d(r\dot{\theta})}{dt} + \dot{r}\dot{\theta} \qquad (12)$$

Using eq.(1), it is easy to see that

$$r\dot{\theta} = V_\theta = V_{TAS} + V_W \operatorname{Cos}\theta \qquad (13)$$

During the On-Pylon Turn maneuver, we will assume both the TAS and the windspeed are constant during the maneuver. Thus, eq. (12) becomes

$$\frac{d(r\dot{\theta})}{dt} + \dot{r}\dot{\theta} = -V_W \operatorname{Sin}\theta\dot{\theta} + V_W \operatorname{Sin}\theta\dot{\theta} = 0 \qquad (14)$$

Equation (14) shows the transverse acceleration to be zero relative to the rotating coordinate system, which is what was stated earlier. It is now clear the radial acceleration relative to the rotating coordinate system is given by

$$(\ddot{r} - r\dot{\theta}^2) = (\frac{dV_r}{dt} - r\dot{\theta}^2) \qquad (15)$$

Since $r\dot{\theta} = V_\theta$, and using eq. (11), eq. (15) becomes

$$(\frac{dV_r}{dt} - r\dot{\theta}^2) = (V_W \operatorname{Cos}\theta\dot{\theta} - V_\theta\dot{\theta}) = -V_{TAS}\dot{\theta} = -\frac{V_{TAS}V_\theta}{r} \qquad (16)$$

The negative sign just indicates that the Centripetal acceleration is directed inward toward the pylon. Here we have used eq. (1) to obtain the final result above.

During the On-Pylon Turn maneuver, the horizontal component of the lift is directed toward the pylon and is equal to the mass times the magnitude of the Centripetal acceleration. Figure 4a shows that the horizontal component of the lift is just equal to $L\sin\phi$, where ϕ is the bank angle. Since the mass of the aircraft is equal to the weight divided by gravitational acceleration g, Newton’s Second Law can be written as

$$\frac{W}{g}\left(\frac{V_{TAS}V_{\theta}}{r}\right) = L\,\mathrm{Sin}\,\phi \quad (17)$$

We can define Centrifugal force as an apparent force with the magnitude equal to the Centripetal force and acting in the opposite direction. In this approach, one allows for an equilibrium balance of forces between the horizontal component of the lift and the Centrifugal force.

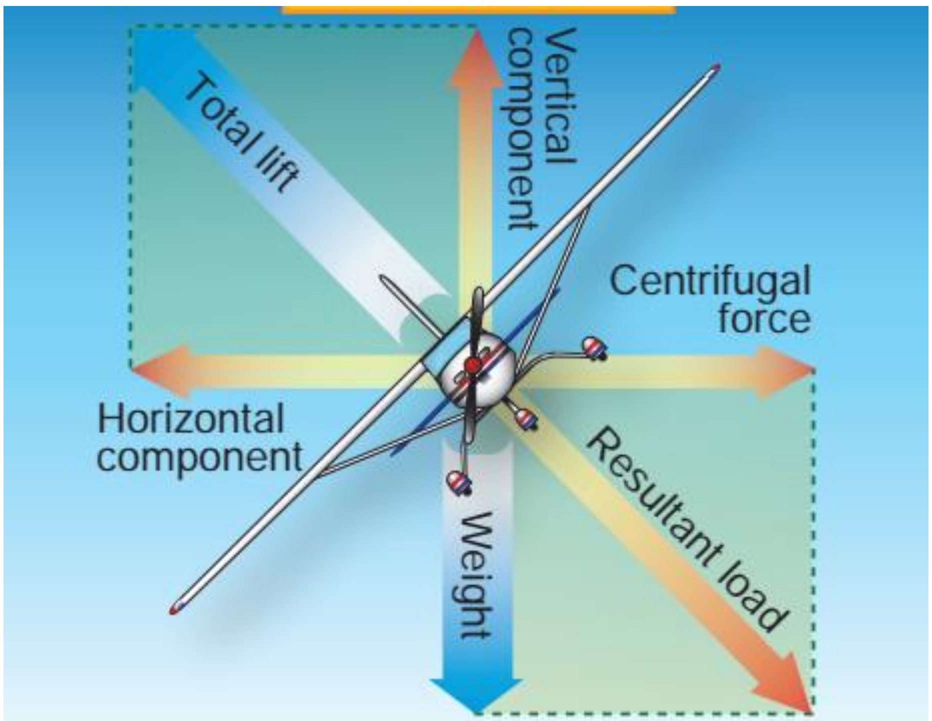

Figure 4a: Balance of Forces in the On-Pylon Turn

(Shallow Flight Path Angle Approximation)

During the On-Pylon Turn maneuver in the presence of a wind, the aircraft is climbing and descending to hold the pylon. Figure 4b shows the balance of forces in the vertical plane during the climbing portion of the maneuver.

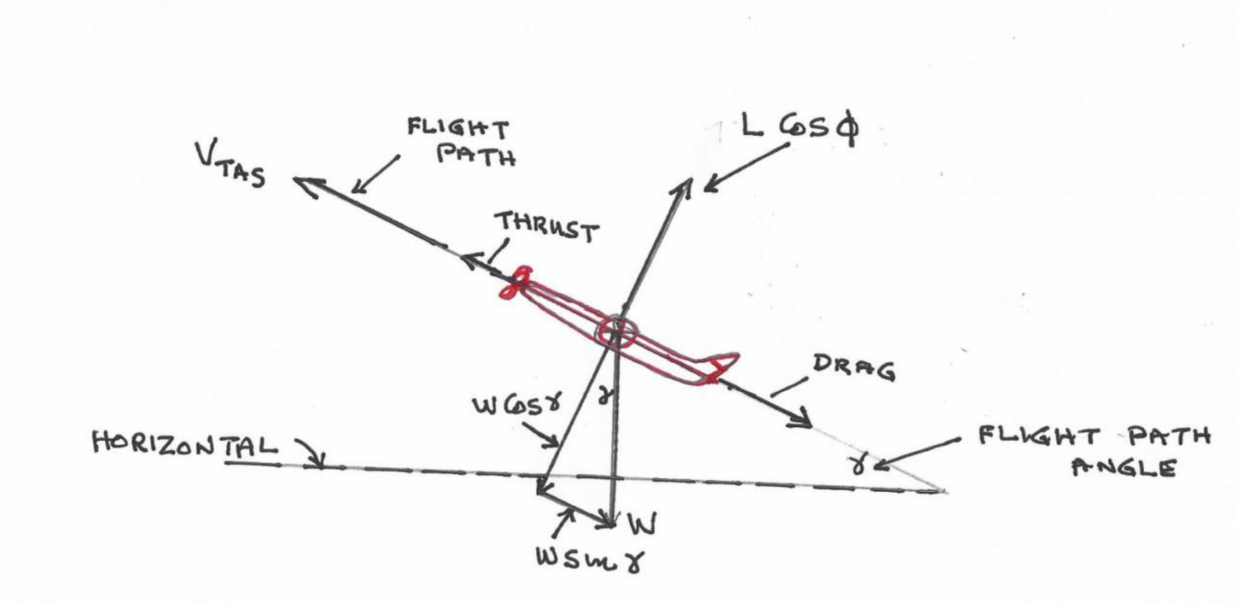

Figure 4b- Balance of Forces in the Vertical Plane during Climbing Flight

Balancing the forces in the vertical direction in Figure 4b, provides us with the following

$$L\,\mathrm{Cos}\,\phi = W\,\mathrm{Cos}\,\gamma \qquad (18)$$

Substituting eq. (18) into eq. (17) gives us the final equation describing the dynamics of the On-Pylon Turn, i.e.

$$\mathrm{Tan}\,\phi = \frac{V_{TAS}V_{\theta}}{gr\cos\gamma} \qquad (19)$$

Where, V_{θ} is given by eq. (13). Finally, since the aircraft must be pivoting on the pylon during the On-Pylon Turn maneuver, the following relationship exists between the aircraft altitude, h, and the radial distance from the pylon, i.e.

$$\frac{h}{r} = \mathrm{Tan}\,\phi \qquad (20)$$

This simple relationship can be seen in Figure 5.

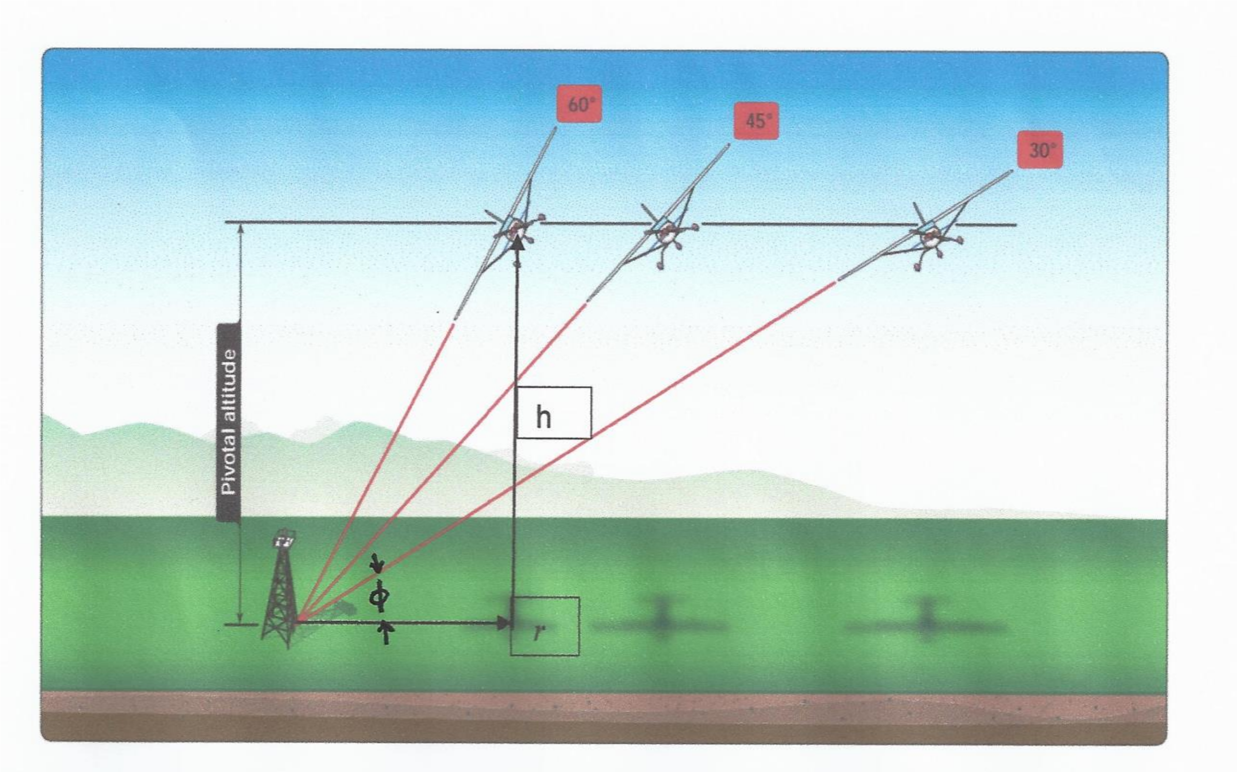

Figure 5: Relationship between the Pivotal Altitude and Distance from Pylon

If we equate the right-hand side of eq. (19) with the left-hand side of eq.(20), we see that the pivotal altitude is given by

$$h = \frac{V_{Tas} V_{\theta}}{g \cos \gamma} \qquad (21)$$

Note that eq.(21) also applies to descending flight, with the sign of the flight path angle going from a positive value to a negative value. In the case of General Aviation aircraft, the flight path angle during this maneuver is usually less than 12-14 degrees for the wind speeds of interest, and thus, the “Shallow Flight Path Angle” approximation can be used, where we can replace the $\cos \gamma$ in eq. (21) with the value unity. With this approximation, eq. (21) reduces to

$$h = \frac{V_{TAS} V_{\theta}}{g} \qquad (22)$$

Although we have assumed the “Shallow Flight Path Angle Approximation”, it will be shown in Section 4.2 by bounding the flight path angle, that this is a valid assumption.

It is best to rewrite eq. (22) as

$$h = \frac{V^2{}_{TAS}}{g}(1+\bar{V}_W \operatorname{Cos}\theta) \qquad (23)$$

Where,

$$\bar{V}_W = \frac{V_W}{V_{TAS}} \qquad (24)$$

Again, we note that the key parameter characterizing the effect of the wind is the windspeed ratio $\bar{V}_W$. It is important to emphasize that V_θ is the ground speed in the transverse direction and does not include the radial component of the wind. It is easy to see that in the case of zero wind, i.e. V_W=0, the pivotal altitude is given by

$$h_{NW} = \frac{V^2{}_{TAS}}{g} \qquad (25)$$

Thus, the pivotal altitude in the presence of a wind can be given in terms of h_{NW} i.e.

$$h = h_{NW}(1+\bar{V}_W \operatorname{Cos}\theta) \qquad (26)$$

In the Airplane Flying Handbook (FAA 8083-3A, 2004), the pivotal altitude in the presence of a wind is given by

$$h_{2004} = \frac{V_{TAS}{}^2}{g} = h_{NW} \qquad (27)$$

Whereas, in the Airplane Flying Handbook (FAA-8083-3B, 2016), the pivotal altitude in the presence of a wind is given by

$$h_{2016} = \frac{V_G{}^2}{g} \qquad (28)$$

Where V_G is defined as the aircraft groundspeed. These results are clearly in error when compared to the correct solution for the pivotal altitude given by eq.(21).

It is easy to see that on the downwind (i.e. $\theta = 0$), the transverse groundspeed is $V_{TAS}(1+\bar{V}_W)$, whereas, on the upwind (i.e. $\theta = 180$), the transverse groundspeed is $V_{TAS}(1-\bar{V}_W)$. When the aircraft is at $\theta = 90$ and $\theta = 270$ degrees, the transverse ground speed is V_{TAS}. Note that $\theta = 90$ and $\theta = 270$ do not correspond to the crosswind points on the ellipse. The crosswind points correspond to values of θ greater than 90 degrees and less than 270 degrees, the actual values depending on the windspeed ratio However, at $\theta = 90$ or $\theta = 270$, the pivotal altitude is $h = h_{NW}$, whereas on the downwind it is $h = h_{NW}(1+\bar{V}_W)$, and on the upwind it is $h = h_{NW}(1-\bar{V}_W)$. The difference in pivotal altitude between the downwind and upwind positions during the On-Pylon Turn maneuver is just

$$\Delta h = 2\bar{V}_W h_{NW} \qquad (29)$$

Equation (29) shows that for a given V_{TAS},

$$\frac{\Delta h}{h_{NW}} = 2\bar{V}_W \qquad (30)$$

Thus, the percent difference in the pivotal altitude between the downwind and upwind position is just twice the windspeed ratio.

The latest version of the Airplane Flying Handbook (FAA-8083-3B) states, "The pivotal altitude is estimated by dividing the square of the groundspeed by 15 (if the airspeed is in miles per hour) or dividing by 11.3 if the groundspeed is in knots". In order to understand the conversion process one should understand that for the pivotal altitude to be in feet, we need to convert V_{TAS} to feet per sec. Note the gravitational constant g=32.174 feet/sec^2. In order to convert the TAS in miles per hour to feet/sec, we multiply the TAS by 1.4667. Thus, the no wind pivotal altitude is just

$$h_{NW} = \frac{V_{TAS}^{2}}{14.956} \qquad (31)$$

Which can be approximated by

$$h_{NW} = \frac{V_{TAS}^{2}}{15} \qquad (32)$$

If the V_{TAS} is in knots, then in order to convert the V_{TAS} to feet/sec, we multiply the V_{TAS} by 1.6875. In this case the no wind pivotal altitude becomes

$$h_{NW} = \frac{V_{TAS}^{2}}{11.298} \qquad (33)$$

Which can be approximated by

$$h_{NW} = \frac{V_{TAS}^{2}}{11.3} \qquad (34)$$

In Section 3 we derive the shape of the track of the aircraft while performing an On-Pylon Turn maneuver.

3.0 Track of the Aircraft during the On-Pylon Turn

In Section 2, we derived the dynamics of the On-Pylon Turn. We showed that the transverse velocity was given by

$$V_\theta = r\frac{d\theta}{dt} = V_{TAS}(1+\bar{V}_W \operatorname{Cos}\theta) \quad (35)$$

In addition, the radial velocity of the aircraft is given by

$$V_r = \frac{dr}{dt} = \dot{r} = V_{TAS}\bar{V}_W \operatorname{Sin}\theta \quad (36)$$

Dividing eq. (36) by eq. (35) we obtain the following equation

$$\frac{\frac{dr}{dt}}{r\frac{d\theta}{dt}} = \frac{1}{r}\frac{dr}{d\theta} = \frac{\bar{V}_W \operatorname{Sin}\theta}{1+\bar{V}_W \operatorname{Cos}\theta} \quad (37)$$

Eq. (37) can be rewritten as

$$\frac{dr}{r} = \frac{(\bar{V}_W \operatorname{Sin}\theta)d\theta}{1+\bar{V}_W \operatorname{Cos}\theta} \quad (38)$$

Using elementary calculus we can integrate eq. (38) which results in the following expression for r, the distance of the aircraft's CG from the pylon, i.e.

$$r = r_0\frac{(1+\bar{V}_W)}{(1+\bar{V}_W \operatorname{Cos}\theta)} \quad (39)$$

Here r_0 is an arbitrary constant which is the distance from the pylon to the aircraft CG when on the downwind. Thus, we see that there are an infinite number of solutions with the same pivotal altitude but with different values of r_0 and associated bank angles. Equation (39) is the equation of an ellipse, with the eccentricity of the ellipse given by the windspeed ratio $\bar{V}_W$. Table 1 shows the distance of the aircraft from the pylon as a function of four angular positions around the pylon. It is easy to see that the aircraft is closest to the pylon when on the downwind ($\theta = 0$), and is farthest from the pylon when on the upwind ($\theta = 180$). At $\theta =$90 and 270 degrees, the distance from the pylon is somewhere between the downwind and upwind distances from the pylon. The ratio of the distance from the pylon on the upwind to the downwind is given by

$$\frac{r(180)}{r_0} = \frac{(1+\bar{V}_W)}{(1-\bar{V}_W)} \quad (40)$$

Thus, for a windspeed ratio of 0.2, we see that the distance from the pylon on the upwind is 50% larger than the distance on the downwind.

Table 1: Distance from the Pylon ($\frac{r}{r_0}$) as a Function of θ

θ (degrees)	r / r_0
0	1
90	$(1+\bar{V}_W)$
180	$\frac{(1+\bar{V}_W)}{(1-\bar{V}_W)}$
270	$(1+\bar{V}_W)$

Figure 6 show the ground track of the aircraft when flying the On-Pylon Turn in the presence of a wind. We show the values of x and y normalized to r_0, the distance from the pylon on the downwind. The ground tracks correspond to four values of the windspeed ratio, $\bar{V}_W$ =0, 0.1, 0.2, and 0.3. These values correspond to the different eccentricity of the ellipse, with $\bar{V}_W = 0$ corresponding to a circle. Note that a windspeed ratio of 0.3 will normally encompass the maximum strength of the wind that the Pilot would encounter while executing the maneuver between 90 and 100 knots TAS, i.e. 27-30 knots.

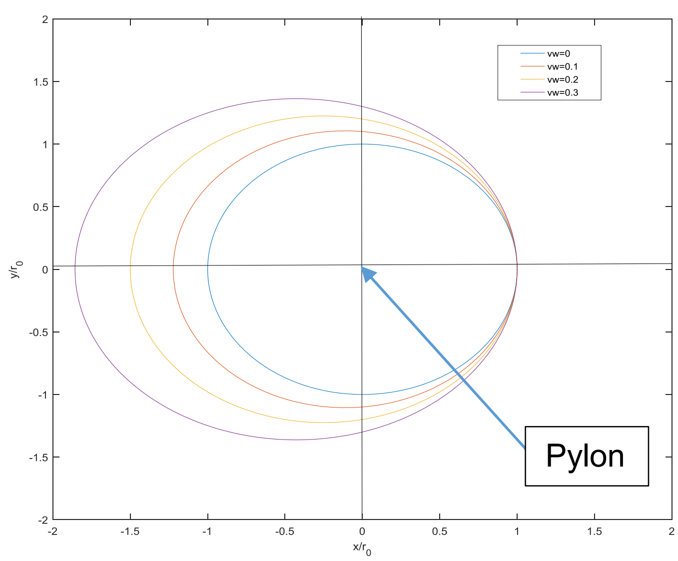

Figure 6: Ground Track around the Pylon for Various $\bar{V}_W$

Figure 7 shows the aircraft ground track corresponding to a windspeed ratio of 0.2. Note that we have designated points A, B, C, and D, which are associated with the downwind, crosswind, upwind and crosswind points. At the upwind and downwind points the aircraft is aligned such that the wind correction angle is identically zero. Thus, the aircraft will attain it maximum wind correction somewhere between points A and C.

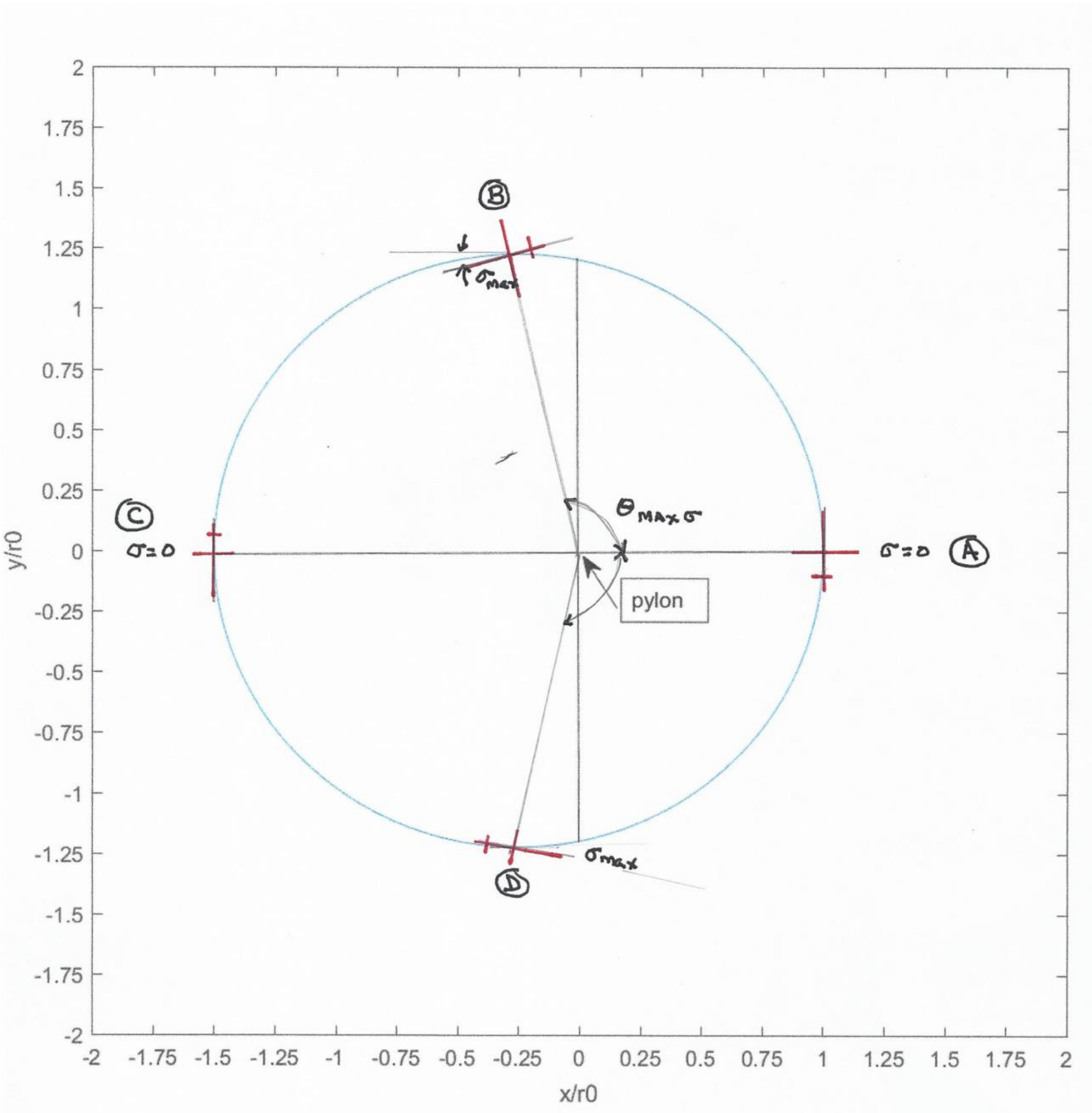

Figure 7: Ground Track for On-Pylon Turn with $\bar{V}_W = 0.2$

At point B the aircraft's track is parallel to the x-axis, so the aircraft must have established a WCA that is consistent with a wind perpendicular to a road aligned with the x-axis. In Ref. 1 we derived the solution to the wind triangle problem, and showed that the WCA was given by

$$\operatorname{Sin}\sigma = \bar{V}_W \operatorname{Sin}\alpha \qquad (41)$$

Where σ is the WCA and α is the angle between the wind direction and the ground track of the aircraft. Since at points B and D, $\alpha = 90$ degrees, the WCA is just given by

$$\sigma = \operatorname{Sin}^{-1}(\bar{V}_W) \qquad (42)$$

In the case of $\bar{V}_W$ =0.2, the WCA is 11.54 degrees.

We will now show that the maximum value of the WCA will always correspond to points B and D. Since the aircraft longitudinal axis is always pointed 90 degrees ahead of the lateral axis (i.e. θ), the direction of the longitudinal axis is given by

$$\theta_{long-axis} = 90 + \theta \qquad (43)$$

The WCA is the angle between the longitudinal axis and the track of the aircraft. The track of the aircraft is given by the direction of the tangent to the ellipse at any value of θ . The tangent to the ellipse at any point is given by

$$\frac{dy}{dx} = \frac{(\frac{dy}{d\theta})}{(\frac{dx}{d\theta})} \qquad (44)$$

One can express x and y as

$$\begin{aligned} x &= r\operatorname{Cos}\theta \\ y &= r\operatorname{Sin}\theta \end{aligned} \qquad (45)$$

Substituting eq. (39) into eq. (45) and then differentiating the result with respect to θ , gives the following expression for the tangent to the ellipse

$$\frac{dy}{dx} = \frac{(\operatorname{Cos}\theta + \bar{V}_W)}{(-\operatorname{Sin}\theta)} \qquad (46)$$

Therefore the angle of the track of ellipse is just

$$\theta_{\tan} = \operatorname{Tan}^{-1}(\frac{dy}{dx}) \qquad (47)$$

Equation (46) is written in this manner in order for $\mathrm{Tan}^{-1}(\frac{dy}{dx})$ to give the correct value of $\theta_{\tan}$.Thus, the WCA is given by

$$WCA = (90 + \theta) - \mathrm{Tan}^{-1}(\frac{dy}{dx}) \qquad (48)$$

If we are interested in the location of the maximum value of the WCA we can differentiate eq.(48) with respect to θ and set the result equal to zero. The roots of this equation provide the location of where the maximum/minimum values of the WCA are located. Performing the differentiation and setting the result to zero, gives the following equation to solve

$$\bar{V}_W(\bar{V}_W + \mathrm{Cos}\,\theta) = 0 \qquad (49)$$

The solution of eq. (49) provides two roots, which are given by

$$\begin{aligned} \bar{V}_W &= 0 \\ \bar{V}_W &= -\mathrm{Cos}\,\theta \end{aligned} \qquad (50)$$

The root $\bar{V}_W = 0$ corresponds to the zero wind case in which the WCA is identically zero and the track of the aircraft is given by a circle. In this case we have located the minimum WCA. This particular case of the On-Pylon Turn corresponds to a Turn around a Point at the pivotal altitude. The second root, $\bar{V}_W = -\mathrm{Cos}\,\theta$, corresponds to the location of the maximum WCA. Note that eq. (46) shows that the value of $\frac{dy}{dx}$ is identically zero when $\bar{V}_W = -\mathrm{Cos}\,\theta$, and therefore point B and D correspond to the maximum WCA. The maximum WCA for point B is located at $\theta = \mathrm{Cos}^{-1}(-\bar{V}_W) = \mathrm{Cos}^{-1}(-0.2) = 101.54$ degrees, and thus, the direction of the aircraft's longitudinal axis is given by $\theta_{long-axis} = 90 + 101.54$ degrees. Since the tangent to the ellipse at this point is oriented at $\theta = 180$ degrees, the WCA is just the difference between 191.54 degrees and 180 degrees, which is 11.54 degrees. The positive sign indicates that the WCA is inward toward the pylon. At point D, the $\mathrm{Cos}^{-1}(-\bar{V}_W) = 258.46$, and thus the direction of the aircraft's longitudinal axis is given by $\theta_{long-axis} = 90 + 258.46$. Since the tangent to the ellipse at this point is oriented at $\theta = 360$ degrees, the WCA is just the difference between 348.46 degrees and 360 degrees, which is -11.54 degrees. The negative sign indicates that the WCA is outward from the pylon. Thus, one can easily determine the magnitude of the maximum WCA during the On-Pylon Turn using eq.(42). When the windspeed ratio is less than 0.5, the maximum WCA in degrees can be estimated by the following equation

$$(WCA)_{\max} = 60\bar{V}_W \qquad (51)$$

It can be seen from Figure 7 that during the On-Pylon Turn, the magnitude of the aircraft's WCA at point A is zero, reaches a maximum at point B, decreases to zero at point C and again reaches its maximum point D.

In Section 4 we will utilize the results of Section 2 and Section 3 to determine the aircraft performance during the On-Pylon Turn.

4.0 Aircraft Performance during the On-Pylon Turn

4.1 Required Turn Rate and Bank Angle

Since the aircraft turn rate is given by eq.(1), we can express the turn rate using eqs. (13) and (39). The resultant turn rate is given by the expression

$$\omega = \dot{\theta} = \frac{V_\theta}{r} = (\frac{V_{TAS}}{r_0})\frac{(1+\bar{V}_W \operatorname{Cos}\theta)^2}{(1+\bar{V}_W)} \qquad (52)$$

When the windspeed ratio is zero, the turn rate becomes

$$\omega_{NW} = \frac{V_{TAS}}{r_0} \qquad (53)$$

Again, r_0 is the distance from aircraft to the pylon at $\theta = 0$, i.e. on the downwind.

Therefore, it is best to introduce the turn rate ratio, which is the ratio of the actual turn rate in a wind to the turn rate with no wind. This results in the following expression for the turn rate ratio

$$\frac{\omega}{\omega_{NW}} = \frac{(1+\bar{V}_W \operatorname{Cos}\theta)^2}{(1+\bar{V}_W)} \qquad (54)$$

Table 2 shows the turn rate ratio corresponding to the identical four points around the ellipse as shown in Table 1

Table 2: Turn Rate Ratio Versus θ

θ (degrees)	$\dfrac{\omega}{\omega_{NW}}$
0	$(1+\bar{V}_W)$
90	$\dfrac{1}{(1+\bar{V}_W)}$
180	$\dfrac{(1-\bar{V}_W)^2}{1+\bar{V}_W}$
270	$\dfrac{1}{(1+\bar{V}_W)}$

Using eqs.(19) and (39), the equation for the bank angle becomes

$$\mathrm{Tan}\,\phi = \left(\frac{V_{TAS}{}^2}{g r_0}\right)\frac{(1+\bar{V}_W\,\mathrm{Cos}\,\theta)^2}{(1+\bar{V}_W)} \qquad (55)$$

If we define the no wind bank angle as

$$\mathrm{Tan}\,\phi_{NW} = \frac{V_{TAS}{}^2}{g r_0} \qquad (56)$$

We can express the bank angle ratio as

$$\frac{\mathrm{Tan}\,\phi}{\mathrm{Tan}\,\phi_{NW}} = \frac{(1+\bar{V}_W\,\mathrm{Cos}\,\theta)^2}{(1+\bar{V}_W)} \qquad (57)$$

Table 3 shows the bank angle ratio at the identical four values of θ shown in Tables 1 and 2. Note that the bank angle ratio and the turn rate ratio are identical, i.e.

$$\frac{\mathrm{Tan}\,\phi}{\mathrm{Tan}\,\phi_{NW}} = \frac{\omega}{\omega_{NW}} \qquad (58)$$

Table 3: Bank Angle Ratio

θ (degrees)	$\dfrac{\mathrm{Tan}\,\phi}{\mathrm{Tan}\,\phi_{NW}}$
0	$(1+\bar{V}_W)$
90	$\dfrac{1}{(1+\bar{V}_W)}$
180	$\dfrac{(1-\bar{V}_W)^2}{1+\bar{V}_W}$
270	$\dfrac{1}{(1+\bar{V}_W)}$

If one is interested in flying a specified maximum bank angle on the downwind, eq. (55) can be used to determine the initial distance from the pylon to the aircraft. As an example, if we want to limit the maximum bank angle to $\phi_{\max}$ on the downwind, the distance from the pylon on the downwind is given by

$$r_0 = \frac{V_{TAS}{}^2}{g\tan\phi_{\max}}(1+\bar{V}_W) \qquad (59)$$

Thus, we see the value of r_0 depends on V_{TAS}, $\bar{V}_W$ and $\phi_{\max}$.

In the next Section, we show how to choose the values of V_{TAS} and $\phi_{\max}$.

4.2 Required Rate of Climb/Descent

When there is no wind, the pivotal altitude will be constant during the On-Pylon Turn maneuver when flown at a constant TAS. However, in the presence of a wind, i.e. $\bar{V}_W \neq 0$, the aircraft will need to descend when travelling from the downwind to the upwind portion of the On-Pylon Turn and will need to climb when travelling from the upwind to the downwind portion of the On-Pylon Turn. Thus, in order to keep the TAS constant during the maneuver, power will need to be adjusted. We will now determine the required rate of climb/descent as a function of the position of the aircraft around the pylon.

We can calculate the required rate of climb or descent by computing the quantity $\frac{dh}{dt}$. The equation for the required pivotal altitude as a function of θ is given by eq. (26). Therefore, we can calculate the rate of climb or descent using the following equation

$$\frac{dh}{dt} = \frac{dh}{d\theta}\frac{d\theta}{dt} \qquad (60)$$

The quantity $\frac{dh}{d\theta}$ can be obtained by differentiating eq. (26) with respect to θ . This results in

$$\frac{dh}{d\theta} = -h_{NW}\bar{V}_W \operatorname{Sin}\theta \qquad (61)$$

Substituting for the rate of turn in eq. (52) into eq.(60), we obtain the following equation for the climb/descent rate

$$\frac{dh}{dt} = -(h_{NW}\bar{V}_W \operatorname{Sin}\theta)[\omega_{NW}\frac{(1+\bar{V}_W \operatorname{Cos}\theta)^2}{(1+\bar{V}_W)}] \qquad (62)$$

The product of h_{NW} and ω_{NW} is given by

$$h_{NW}\omega_{NW} = (\frac{{V_{TAS}}^2}{g})(\frac{V_{TAS}}{r_0}) = \frac{{V_{TAS}}^3}{gr_0} \qquad (63)$$

Thus, eq. (62) becomes

$$\frac{dh}{dt} = -(\frac{{V_{TAS}}^3}{gr_0})(\frac{\bar{V}_W}{1+\bar{V}_W})[\operatorname{Sin}\theta(1+\bar{V}_W \operatorname{Cos}\theta)^2] \qquad (64)$$

Note that at θ =0 and 180 degrees, the $\operatorname{Sin}\theta$ =0 and $\frac{dh}{dt}$ =0. At these points around the pylon, the aircraft is reaching the maximum and minimum pivotal altitudes, respectively. At θ =90 and 270 degrees, $\frac{dh}{dt}$ becomes

$$\frac{dh}{dt} = \mp\frac{V^3{}_{TAS}}{gr_0}(\frac{\bar{V}_W}{1+\bar{V}_W}) \qquad (65)$$

Where the negative sign corresponds to θ =90 degrees and the positive sign corresponds to θ =270 degrees. Since the units of $\frac{dh}{dt}$ in eq.(64) are in feet/sec, we need to multiply the right hand side by 60 to obtain the rate of climb/descent in feet/min. This results in the following equation for $\frac{dh}{dt}$

$$\frac{dh}{dt} = -60(\frac{V_{TAS}{}^3}{gr_0})(\frac{\bar{V}_W}{1+\bar{V}_W})[\mathrm{Sin}\,\theta(1+\bar{V}_W\,\mathrm{Cos}\,\theta)^2] \qquad (66)$$

If one is interested in the maximum required climb or descent rate during the On-Pylon Turn maneuver, we can find the maximum or minimum value by differentiating eq. (66) with respect to θ and setting the result equal to zero. One can then solve the equation for the values of θ corresponding to the maximum value of the rate of climb and descent. Differentiating eq. (66) with respect to θ and setting the result to zero gives the following equation

$$(\mathrm{Cos}\,\theta)_{Max} = \frac{-1+\sqrt{1+24\bar{V}_W^2}}{6\bar{V}_W} \qquad (67)$$

Therefore, the location of the maximum rate of climb/descent is given by

$$(\theta)_{\max} = \mathrm{Cos}^{-1}(\frac{-1+\sqrt{1+24\bar{V}_W^2}}{6\bar{V}_W}) \qquad (68)$$

Since the value of $\mathrm{Cos}\,\theta_{\max} \geq 0$, the maximum lies in the first and fourth quadrants. This means that the maximum rate of descent will occur between θ =0 and 90 degrees, and the maximum rate of climb will occur between θ =270 and 360 degrees. A rough gauge on the maximum rate of climb/descent can be found by evaluating eq. (66) at $\theta = 90$ and 270 degrees, i.e.

$$(\frac{dh}{dt})_{\max} \approx \mp 60(\frac{V_{TAS}{}^3}{gr_0})(\frac{\bar{V}_W}{1+\bar{V}_W}) \qquad (69)$$

In order to choose the value of V_{TAS} which does not exceed the aircraft climb performance, we can substitute the expression for r_0 in eq. (59) into the right hand side of eq.(66), which results in the following equation for $\frac{dh}{dt}$, i.e.

$$\frac{dh}{dt} = -60V_{TAS}\tan\phi_{\max}\frac{\bar{V}_W}{(1+\bar{V}_W)^2}[\mathrm{Sin}\,\theta(1+\bar{V}_W\,\mathrm{Cos}\,\theta)^2] \qquad (70)$$

Evaluating eq. (70) at θ =90 and 270 degrees gives the following approximation for the maximum rate of climb/descent

$$(\frac{dh}{dt})_{\max} \approx \mp 60V_{TAS}\tan\phi_{\max}\frac{\bar{V}_W}{(1+\bar{V}_W)^2} \qquad (71)$$

Note that the magnitude of the maximum rate of climb/descent is proportional to V_{TAS} and the maximum bank angle on the downwind, i.e. $\phi_{\max}$. Thus, the use of higher values of V_{TAS} and larger bank angles on the downwind leads to higher required rates of climb/descent. It is easy to understand these results, since larger values of both bank angles and V_{TAS} lead to higher turn rates, which translate to a shorter time required for a given change in the pivotal altitude from the downwind to the upwind and from the upwind to the downwind.

Equation (71) can be written in the following non-dimensional form

$$H = \frac{(\frac{dh}{dt})_{\max}}{V_{TAS}\tan\phi_{\max}} = \mp 60\frac{\bar{V}_W}{(1+\bar{V}_W)^2} \qquad (72)$$

In this form, the right hand side is only a function of the windspeed ratio. Table 4 below, shows the values of the magnitude of H corresponding to values of $\bar{V}_W$ = 0.1, 0.2, and 0.3.

Table 4: Absolute Values of H versus $\bar{V}_W$

$\bar{V}_W$	$\lvert H \rvert$
0.1	4.959
0.2	8.333
0.3	10.65

As an example if, V_{TAS}=90 Kts, $\phi_{\max}$ =45 degrees, and $\bar{V}_W = 0.1$, eq. (71) gives the maximum required rate of climb/descent as 753.1 ft/min, whereas, using eqs.(68) and (70), gives the maximum rate of climb/descent as 767.7 feet/min. Here we have utilized the fact that the tangent of 45 degrees is unity. Note that at larger values of $\bar{V}_W$, i.e. 0.3, the error between eq.(70) and eq. (71), is approximately 13%. However, eq. (71) should be a suitable approximation for checking to see whether the required aircraft rate of climb performance can be met.

Table 5 shows the magnitude of maximum required climb/descent rate obtained from eq. (71) for the case V_{TAS}=90 Kts and $\phi_{\max} = 45$ degrees. It is easy to see that when $\bar{V}_W \geq$ 0.2, the required maximum rate of climb necessary to hold the pylon would be impossible to achieve in a C-172. When the windspeed ratios reach these levels, one would need to either reduce the TAS, reduce the maximum bank angle, or a combination of both.

Table 5: Magnitude of Maximum Rate Climb/Descent from Eq. (71)

(V_{TAS} =90 Kts, $\phi_{\max} = 45$ deg)

$\bar{V}_W$	$\left\|(\frac{dh}{dt})_{\max}\right\|$ (ft/min)
0.1	753.1
0.2	1265.6
0.3	1617.6

Figure 8 shows a plot of eq. (70) for the required rate of climb/descent as a function of angular position around the pylon. Again, we have taken V_{TAS}=90 Kts and $\phi_{\max} = 45$ degrees. Note that as the windspeed ratio increases, the required rate of climb increases, and can easily exceed the performance capability of the aircraft. Using a value of V_{TAS}=90, the maximum required rate of climb is 1861 feet/min, which is beyond the capability of a C-172. Therefore, the Pilot/Instructor should use eq.(71) to determine the appropriate combination of V_{TAS} and $\phi_{\max}$ such that the aircraft will be able to meet the climb requirements for the On-Pylon Turn. Once these two parameters are selected, the downwind distance from the pylon r_0 can be determined using eq. (59). As an example, with a windspeed ratio of 0.1 at V_{TAS}=90 Kts and a 45 degree bank angle on the downwind, requires the aircraft to be 788.6 feet from the pylon while on the downwind. It is clear from Figure 8, that there is a continuous change in pivotal altitude during the maneuver.

Recall that we have utilized the “Shallow Flight Path Angle” approximation in the above analysis. We can validate this assumption by calculating the flight path angle at the peak value of the rate of climb or descent. The aircraft rate of climb/descent is given by

$$\frac{dh}{dt} = V_{TAS} \sin\gamma \qquad (73)$$

Solving for the $\sin\gamma$ we obtain the following equation

$$\sin\gamma = \frac{1}{V_{TAS}} \frac{dh}{dt} \qquad (74)$$

As an example, in the case of a windspeed ratio of 0.3 with the aircraft flying at V_{TAS}=90 Kts with a maximum bank angle of 45 degrees on the downwind, the required maximum rate of climb/descent is 1861 feet per minute. Dividing this value by the TAS in feet per minute (V_{TAS} =9112.5) gives the value 0.204 for $\mathrm{Sin}\,\gamma$. Taking the inverse Sine of 0.204 shows the maximum flight path angle to be 11.77 degrees. Since

$Cos(11.77) = 0.979$, which is very close to unity, thus validating the "Shallow Flight Path Angle" approximation for the On-Pylon Turn analysis.

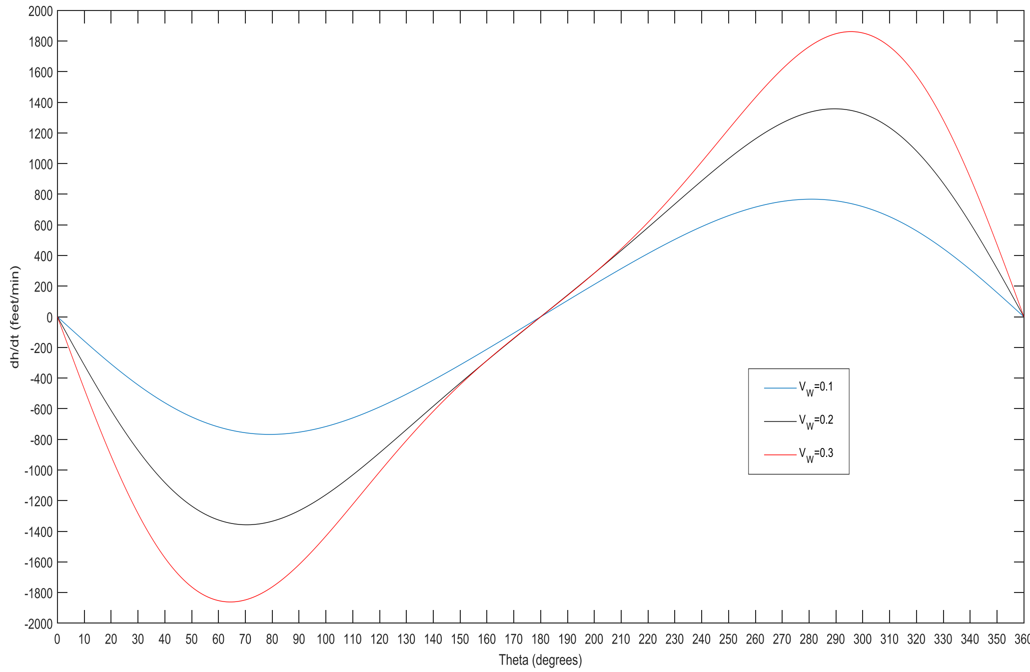

Figure 8: Required Rate of Climb/Descent for the On-Pylon Turn versus θ
(V_{TAS} =90, ϕ_{max} =45 degrees)

In Section 5 we discuss an innovative approach to flying the On-Pylon Turn maneuver, where rather than climbing or descending to hold the pylon, the Pilot utilizes power to control the TAS in order to hold the pylon. In this approach, the aircraft will be flying at a constant pivotal altitude while performing the On-Pylon Turn maneuver.

5.0 Flying the On-Pylon Turn in a Wind at Constant Pivotal Altitude by Utilizing Power to Vary the TAS

Let us consider the case of flying the On-Pylon Turn in the presence of a wind. If we allow the TAS to vary, then we can rewrite eq.(23), i.e.

$$h = \frac{V_{TAS}(V_{TAS} + V_W \cos\theta)}{g} \qquad (75)$$

We will assume the aircraft enters the On-Pylon Turn on the downwind ($\theta = 0$), with a TAS given by $V_{TAS} = V_{T_0}$. If we normalize V_{TAS} and V_W by V_{T_0} , i.e.

$$\begin{aligned} \bar{V}_{TAS} &= \frac{V_{TAS}}{V_{T_0}} \\ \bar{V}_W &= \frac{V_W}{V_{T_0}} \end{aligned} \qquad (76)$$

We can express the pivotal altitude by

$$h = \left(\frac{V_{T_0}^{\ 2}}{g}\right)\bar{V}_{TAS}(\bar{V}_{TAS} + \bar{V}_W \cos\theta) \qquad (77)$$

Where $h_{NW} = \left(\frac{V_{T_0}^{\ 2}}{g}\right)$ and the pivotal altitude on the downwind is given by

$$h(0) = \left(\frac{V_{T_0}^{\ 2}}{g}\right)(1 + \bar{V}_W) \qquad (78)$$

It is easy to see that if we are interested in maintaining a constant pivotal altitude given by eq.(78), the normalized TAS $\bar{V}_{TAS}$ must satisfy eq. (77). Substituting the right hand side of eq. (78) for the left hand side of eq.(77), gives the following quadratic equation for $\bar{V}_{TAS}$

$$\bar{V}_{TAS}^2 + (\bar{V}_W \cos\theta)\bar{V}_{TAS} - (1 + \bar{V}_W) = 0 \qquad (79)$$

The solution of eq. (79) gives the following expression for $\bar{V}_{TAS}$, i.e.

$$\bar{V}_{TAS} = \frac{-\bar{V}_W \cos\theta + \sqrt{4(1 + \bar{V}_W) + (\bar{V}_W \cos\theta)^2}}{2} \qquad (80)$$

Equation (80) gives the required variation of the TAS as a function of the angular position θ, with the key parameter being the windspeed ratio $\bar{V}_W$. Figure 9 shows the required variation of the normalized TAS to hold the pylon versus θ, for windspeed ratios given by 0.1, 0.2, and 0.3. It is easy to see that with the exception of the upwind and downwind locations, the variation in the normalized TAS is nearly linear. Equation (80) shows that the value of $\bar{V}_{TAS}$ on the downwind is unity, while on the upwind, it is $(1+\bar{V}_W)$. Thus approximately half the required increase in TAS occurs in the first 90 degrees of turn, and the remaining half is required in the second 90 degrees. During the second half of the maneuver, the value of $\bar{V}_{TAS}$ will decrease back to unity, with half of the decrease occurring by θ =270 and the remaining half by θ =360. Note that the peak value of $\bar{V}_{TAS}$ provides the information on how much the TAS will need to change from downwind to upwind. It can be seen that the required increase in the TAS at the upwind point is approximately equal to $\bar{V}_W$. As an example, in the case of $\bar{V}_W$ =0.1, the TAS will need to increase by approximately ten percent between the downwind and the upwind.

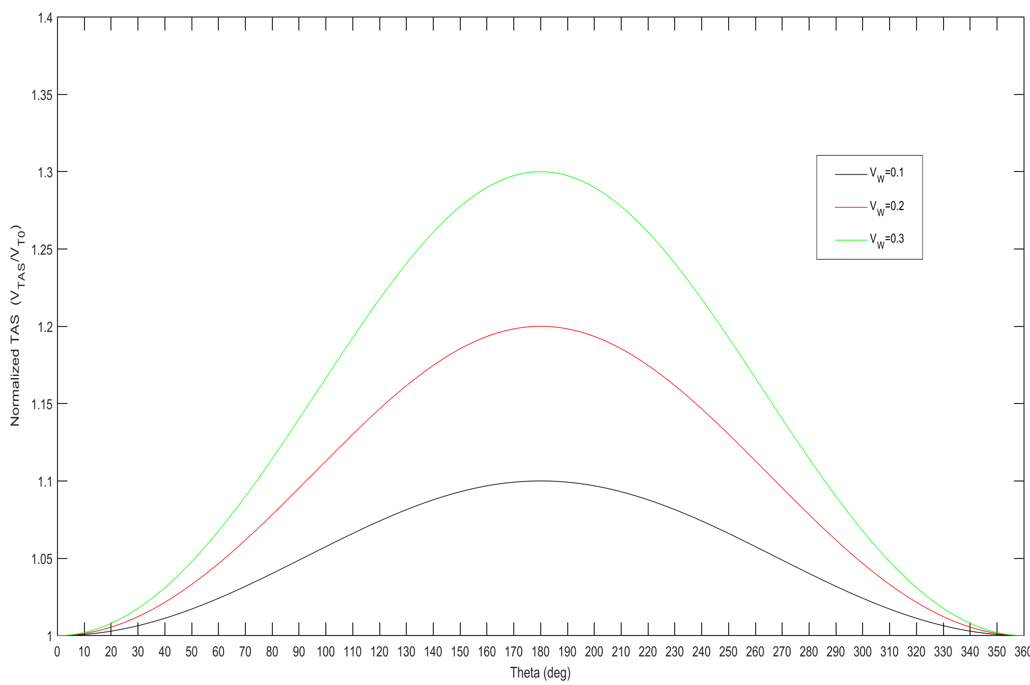

Figure 9: Normalized TAS versus θ ($\bar{V}_W$ =0.1, 0.2, and 0.3)

In order to determine the bank angle and rate of turn for the variable TAS method of holding the pylon, we require the distance from the pylon as a function of θ. Using eq.(38), where in the denominator we have replaced the 1 by $\bar{V}_{TAS}$, and with $\bar{V}_W = \frac{V_W}{V_{T_0}}$, we obtain the following differential equation for r,

$$\frac{dr}{r} = \frac{\bar{V}_W \sin\theta d\theta}{\bar{V}_{TAS} + \bar{V}_W \cos\theta} \qquad (81)$$

Substituting $\bar{V}_{TAS}$ from eq. (80) into eq.(81), and integrating from $\theta = 0$ to an arbitrary value of θ , gives the following equation for $\frac{r}{r_0}$

$$\frac{r}{r_0} = \frac{2(1+\bar{V}_W)e^{-\xi}}{[\bar{V}_W \cos\theta + \sqrt{4(1+\bar{V}_W) + (\bar{V}_W \cos\theta)^2}\,]} \qquad (82)$$

Where ξ is given by

$$\xi = \frac{\{\bar{V}_W \cos\theta[-\bar{V}_W \cos\theta + \sqrt{4(1+\bar{V}_W) + (\bar{V}_W \cos\theta)^2}\,] - 2\bar{V}_W\}}{4(1+\bar{V}_W)} \qquad (83)$$

Using eqs.(82) and (83), we can obtain the following equations for the turn rate ratio and the bank angle ratio, i.e.,

$$\frac{\omega}{\omega_{NW}} = \frac{(\bar{V}_{TAS} + \bar{V}_W \cos\theta)}{r/r_0} \qquad (84)$$

$$\frac{\tan\phi}{\tan\phi_{NW}} = \frac{\bar{V}_{TAS}(\bar{V}_{TAS} + \bar{V}_W \cos\theta)}{(r/r_0)} \qquad (85)$$

Where

$$\omega_{NW} = \frac{V_{T_0}}{r_0}$$
$$\tan\phi_{NW} = \frac{V_{T_0}^2}{g r_0} \qquad (86)$$

In the constant TAS case, the ratio of the bank angle ratio to the turn rate ratio was unity. However, in the variable TAS case, if we divide eq. (85) by eq. (84), we see that this ratio κ is given by

$$\kappa = \frac{[\frac{\tan\phi}{\tan\phi_{NW}}]}{[\frac{\omega}{\omega_{NW}}]} = \bar{V}_{TAS} \geq 1 \qquad (87)$$

Thus, for a given turn rate ratio, the bank angle ratio is larger when the Pilot holds the pylon by changing the TAS, compared to the method of holding the pylon by changing altitude. Figure 10 show a comparison of the normalized distance from the pylon for both the constant TAS and variable TAS methods (i.e. constant pivotal altitude), for values of $\bar{V}_W$ =0.1, 0.2, and 0.3. Here we have assumed a value of V_{T_0} =90 Kts. Note that in the constant pivotal altitude case, the normalized distance from the pylon between θ equals 90 and 270 degrees has decreased. Since the distance from the pylon has decreased in this range of θ , one would expect the corresponding normalized turn rate and bank angle to be increased in the constant pivotal altitude method. However, Figures 11 and 12 show significant changes in the turn rate ratio and bank angle ratio over a large portion of the On-Pylon Turn. This is due to the increase in the TAS as the aircraft moves from the downwind to the upwind, and then the corresponding decrease in airspeed as the aircraft moves from the upwind back to the downwind. However, the bank angle during the On-Pylon Turn will always be bounded by the bank angle on the downwind. Thus, the method of varying the TAS to hold the pylon appears to be viable as an alternative method to that shown in the latest FAA Airplane Flying Handbook (Ref. 3).

In Section 6, we will discuss flying the On-Pylon Turn, including what visual cues the Pilot will observe when he/she is above or below the pivotal altitude, and techniques used to recapture the pylon.

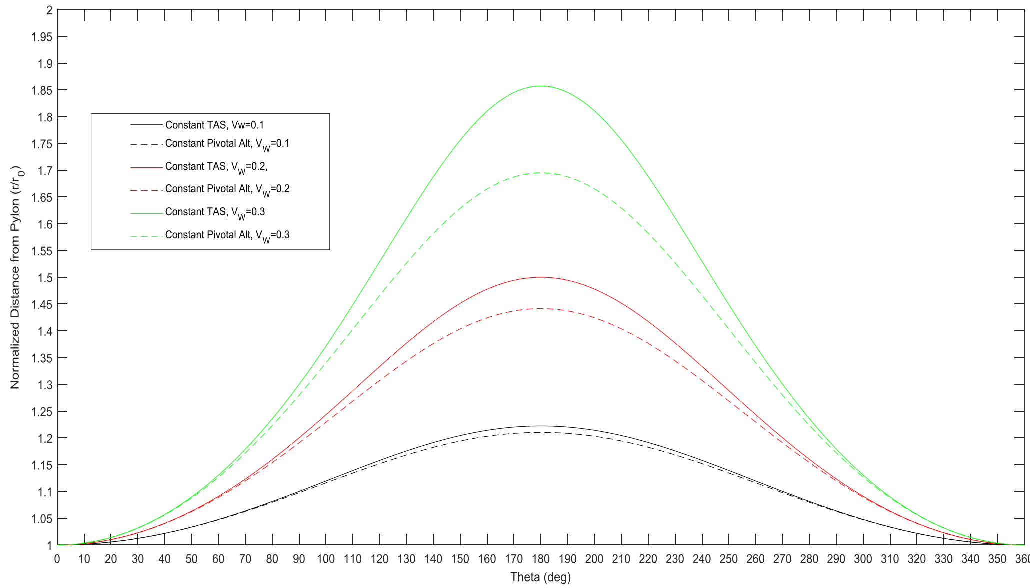

Figure 10: Comparison of Normalized Distance from Pylon $\frac{r}{r_0}$ versus θ

($\bar{V}_W$ =0.1, 0.2, and 0.3)

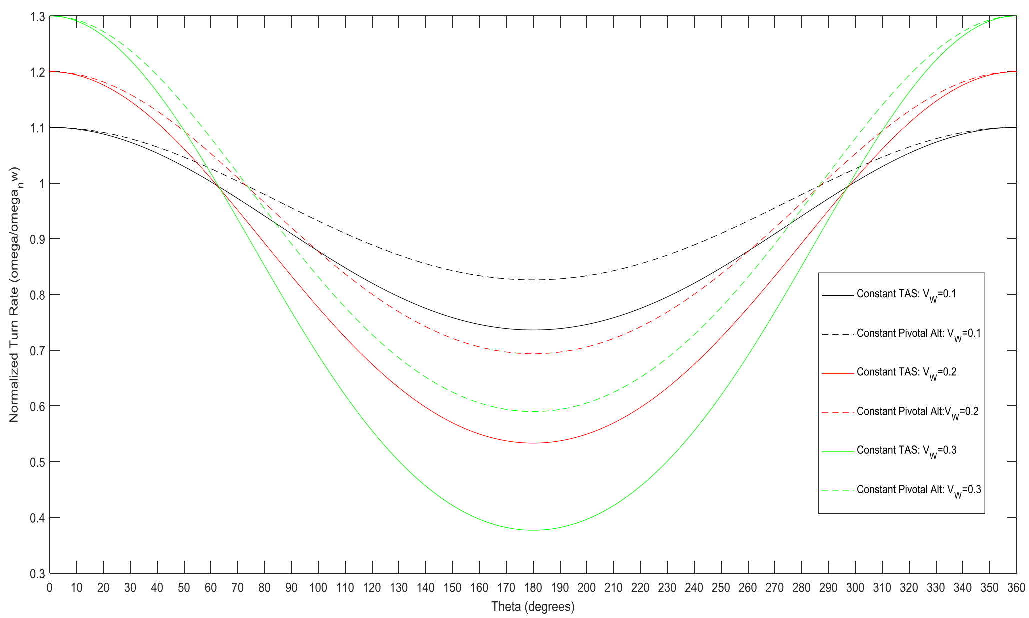

Figure 11: Comparison of Normalized Turn Rate versus θ ($\bar{V}_W$ =0.1, 0.2, and 0.3)

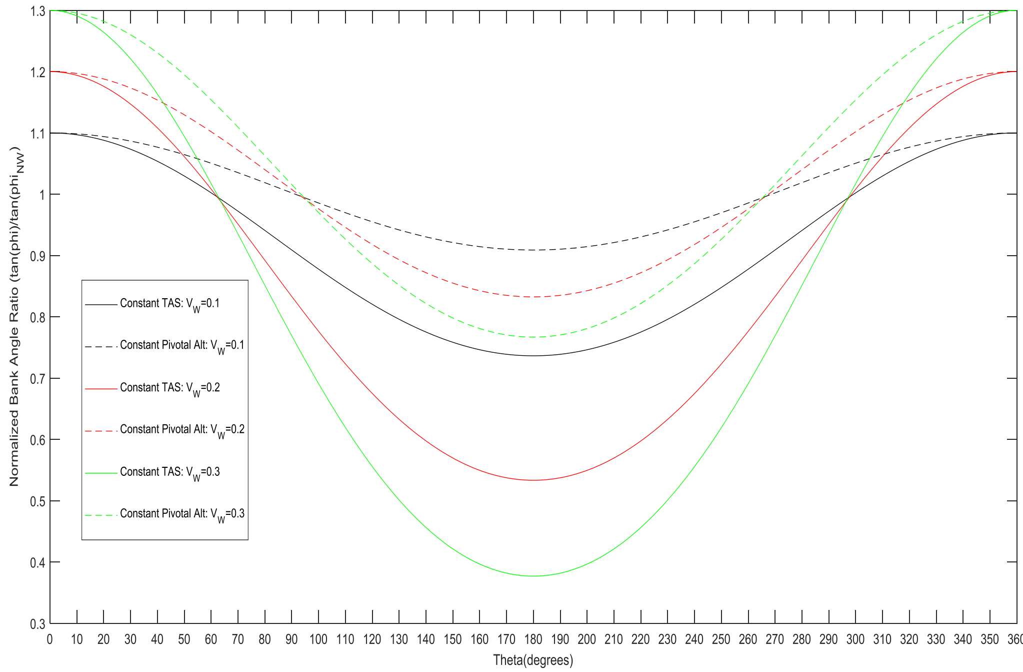

Figure 12: Comparison of Bank Angle Ratio versus θ ($\bar{V}_W$ =0.1, 0.2, and 0.3)

6.0 Flying the On-Pylon Turn

Prior to flying the on-pylon turn, the Pilot/Instructor should prepare for the execution of the maneuver by determining the V_{TAS} and distance from the pylon on the downwind (r_0), that will be used based on the current wind conditions, in order that the aircraft will be able to perform the required rate of climb for the maneuver. Equations (59) and (71) can be used to determine these parameters. In addition, if the maneuver will be performed at higher elevations above MSL, the Pilot must insure that he/she is using TAS and not IAS,. The Pilot/Instructor should also determine the pivotal altitudes for the four points around the pylon shown in Table 4. Since pivotal altitude is relative to the surface, it should be added to the elevation of the surface, so as to obtain the indicated altitude of the aircraft at those points.

Table 4: Ratio of $\frac{h}{h_{NW}}$ as a Function the θ

θ (degrees)	$\frac{h}{h_{NW}}$
0	$(1+\bar{V}_W)$
90	1
180	$(1-\bar{V}_W)$
270	1

Again, the pivotal altitude in the no wind case is given by eq.(25), i.e. $h_{NW} = \frac{V_{TAS}{}^2}{g}$. With this information available, it is anticipated that the Pilot/Instructor will be able to stay ahead of the aircraft during the execution of this maneuver.

We will now shift our attention to developing a technique for determining whether the aircraft is above or below the pivotal altitude. In order to facilitate this discussion we will consider the no wind case, i.e. $\bar{V}_W = 0$. Once the no wind case is understood, the technique can easily be carried over to the case where $\bar{V}_W \neq 0$.

6.1 No Wind Case

In Section 3 we derived the track of the aircraft during the On-Pylon Turn maneuver. It was shown that the shape of the track is that of an ellipse with the eccentricity given by the parameter $\bar{V}_W$. In the no wind case, the track of the aircraft becomes that of a circle of constant radius. Thus, the On-Pylon Turn is also a Turn around a Point, with the point being the pylon. As shown earlier, there are an infinite number of solutions to the On-Pylon Turn maneuver, corresponding to different radii/bank angles, with the radius and bank angle related by eq.(56), i.e. $\mathrm{Tan}\,\phi_{NW} = \frac{V_{TAS}{}^2}{g r_0}$.
If the Pilot enters the On-Pylon Turn at a selected value of r_0, and then rolls into a bank angle given by eq.(56), the aircraft will execute a perfect Turn around a Point. Unless the aircraft is at the pivotal altitude, the reference line will not appear to pivot on the pylon. Figure 13 shown below, taken from the latest Airplane Flying Handbook (FAA-H-8083-3B, 2016), attempts to convey the message that if the aircraft is below the pivotal altitude, the projection of the reference line onto the surface will scribe a circle in the forward direction. If the aircraft is above the pivotal altitude, the projection of the reference line onto the surface will scribe out a circle in the reverse direction.

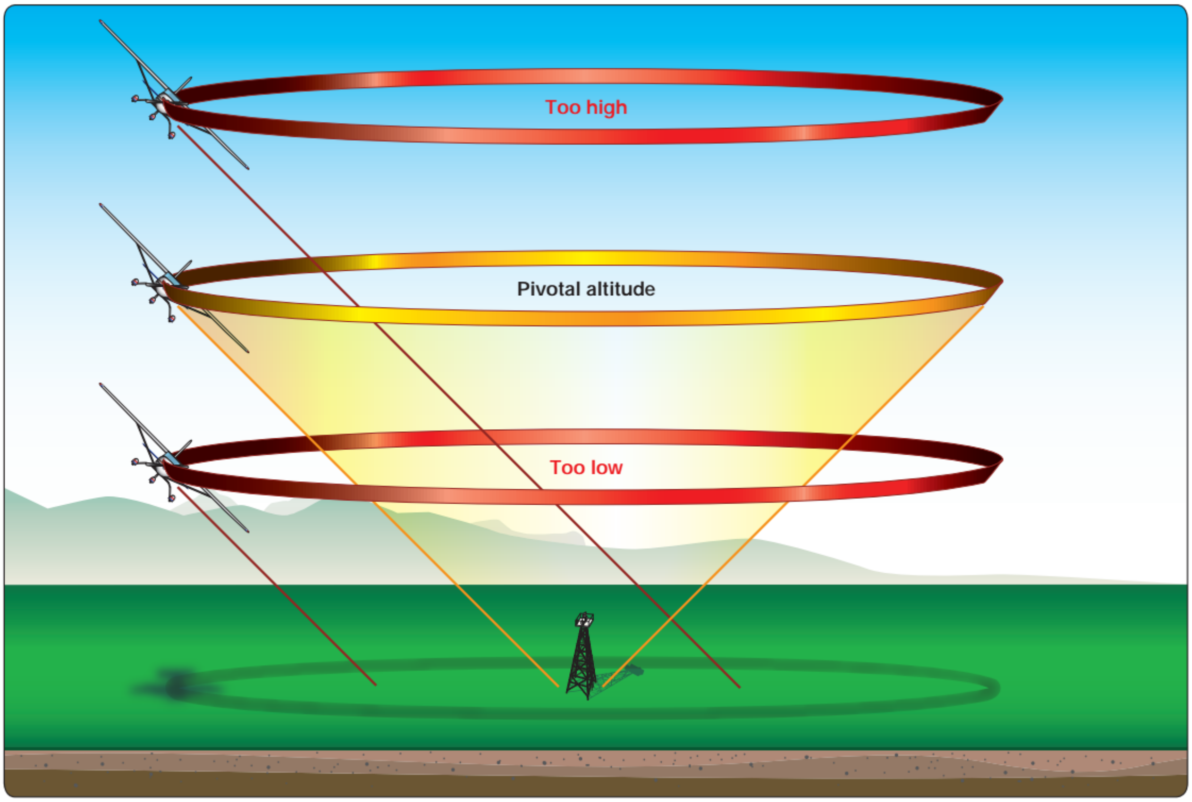

Figure 13: Projection of Reference Line on Surface for Aircraft Maintaining a Constant Bank Angle and Radius

For those not able to grasp the above explanation, a more physical explanation is shown below in Figure 14.

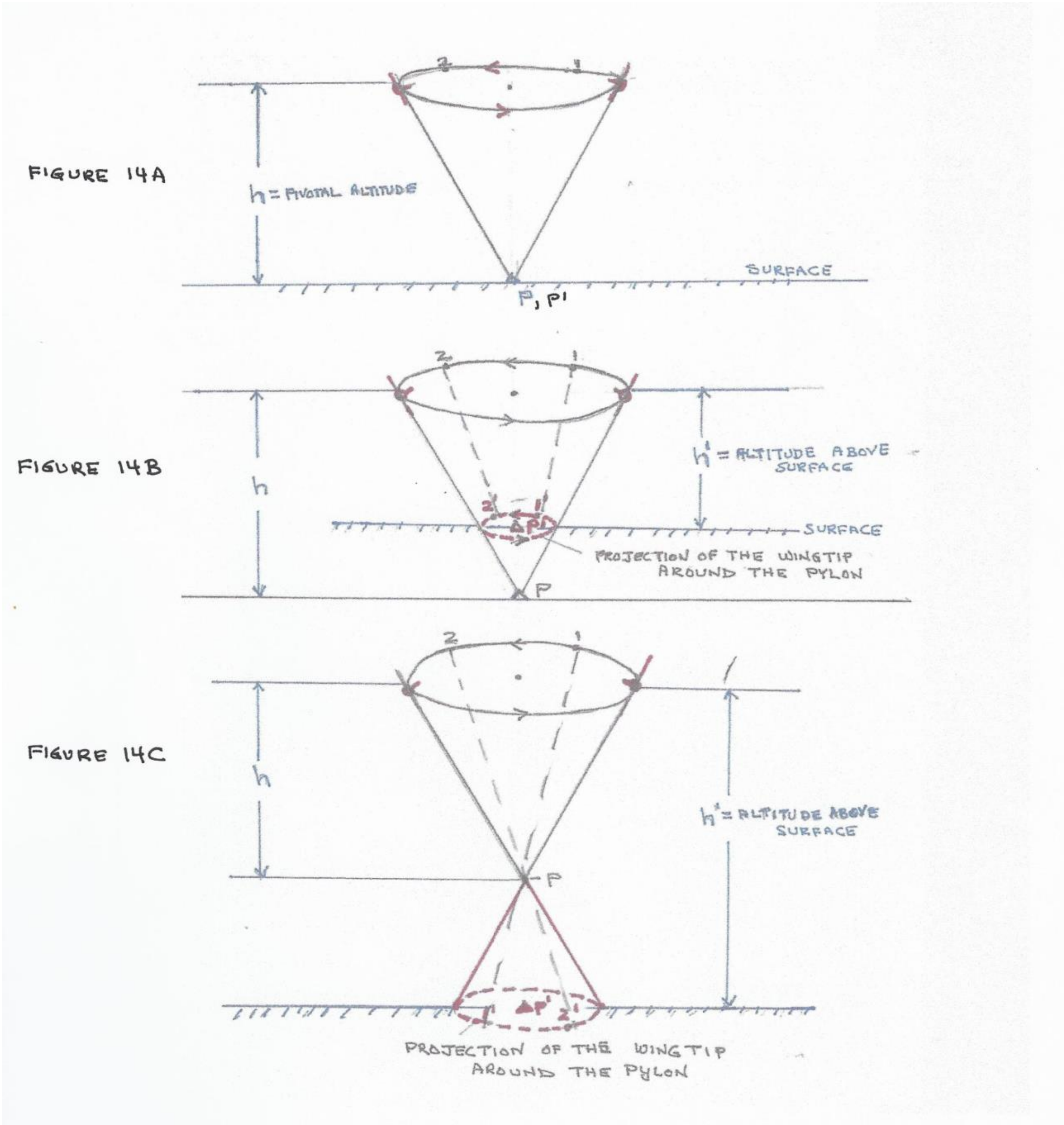

Figure 14: Visual Cues for Determining Correct Pivotal Altitude

In this explanation, no matter what altitude the aircraft is flying, if one places an imaginary pylon at a distance $h=h_{nw}$ below the aircraft, the reference line will always appear to pivot on the imaginary pylon. Let's assume that the actual pylon is located at the point P', and the imaginary pylon is located at the point P. Consider Figure 14A, where the aircraft is flying at an altitude corresponding to the pivotal altitude. In this case the points P and P' are identical, and the reference line will appear to pivot on the pylon. In Figure 14B, the aircraft is flying at a height h', which is below the pivotal altitude. We see that the reference line appears to pivot on P, while the projection of the aircraft track onto the surface appears to move forward in a circle. Note that the points 1 and 2 project to points 1' and 2' respectively. Figure 14C depicts the case when the aircraft is above the pivotal altitude. Note that in this case the circular cone is reflected below the surface, with the apex being the point P. Again, points 1 and 2 project to points 1' and 2' respectively. In this case, the projection of the reference line on the surface appears to move backwards.

In the general scenario, if the aircraft is not at the pivotal altitude, the bank angle that initially places the reference line on the pylon will not correspond to the radius of the turn at the pivotal altitude. In Figure 15, we show the conic surface representing a particular bank angle.

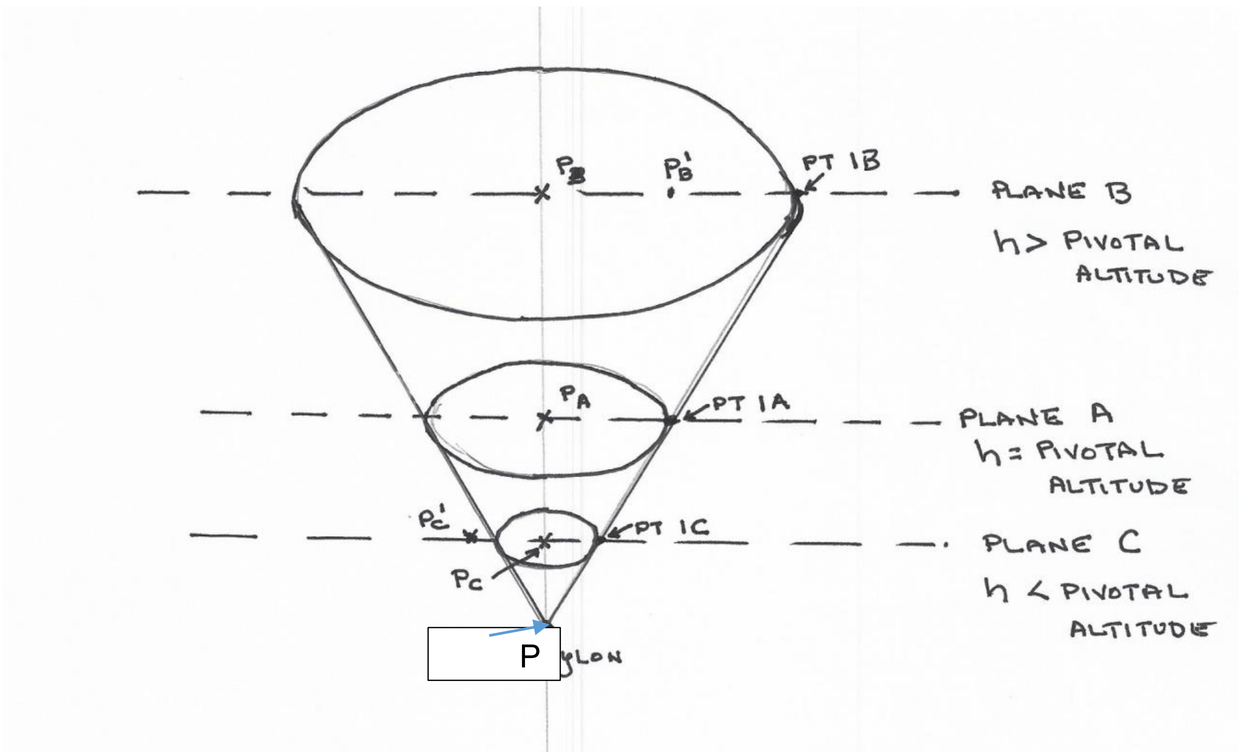

Figure 15: Aircraft Initiating the On-Pylon Turn from Three Different Altitudes

(Plane A at Pivotal Altitude, Plane B above Pivotal Altitude, and Plane C below Pivotal Altitude)

Here we see that if the aircraft has initiated the On-Pylon Turn on plane A at point 1A, the reference line will appear to pivot on the pylon, i.e. point P. If the aircraft has initiated it turn on plane B at point 1B (i.e. above the pivotal altitude), the aircraft will be flying the same radius as on plane A, since the bank angle would be the same. This causes the center of the turn to move closer to the aircraft on Plane B, i.e. P'_B. Figure

16 shows the resultant movement of the reference line relative to the pylon (P). We see that as the aircraft moves from position 1 to 2, 3, and 4, the reference line is moving rearward relative to the pylon located a point P, i.e. points 1', 2', 3', and 4'. This is the indication that the aircraft is above the pivotal altitude and needs to descend and move to a smaller radius. As the altitude is reduced, the reference line will start to reverse direction and move toward the pylon. When the aircraft finally reaches the pivotal altitude, the reference line will have caught up to the pylon. However, the bank angle will no longer correspond to the initial bank angle at point 1A, it will have changed in order to keep the elevation of the reference line on the pylon. Note that if the initial bank angle is kept constant during the descent to the pivotal altitude, the aircraft will appear to pivot on the point which is the projection of P'_B onto the surface, rather than the point P. Thus to maintain the original pylon at point P, the aircraft bank will be varying slightly during the descent to the pivotal altitude.

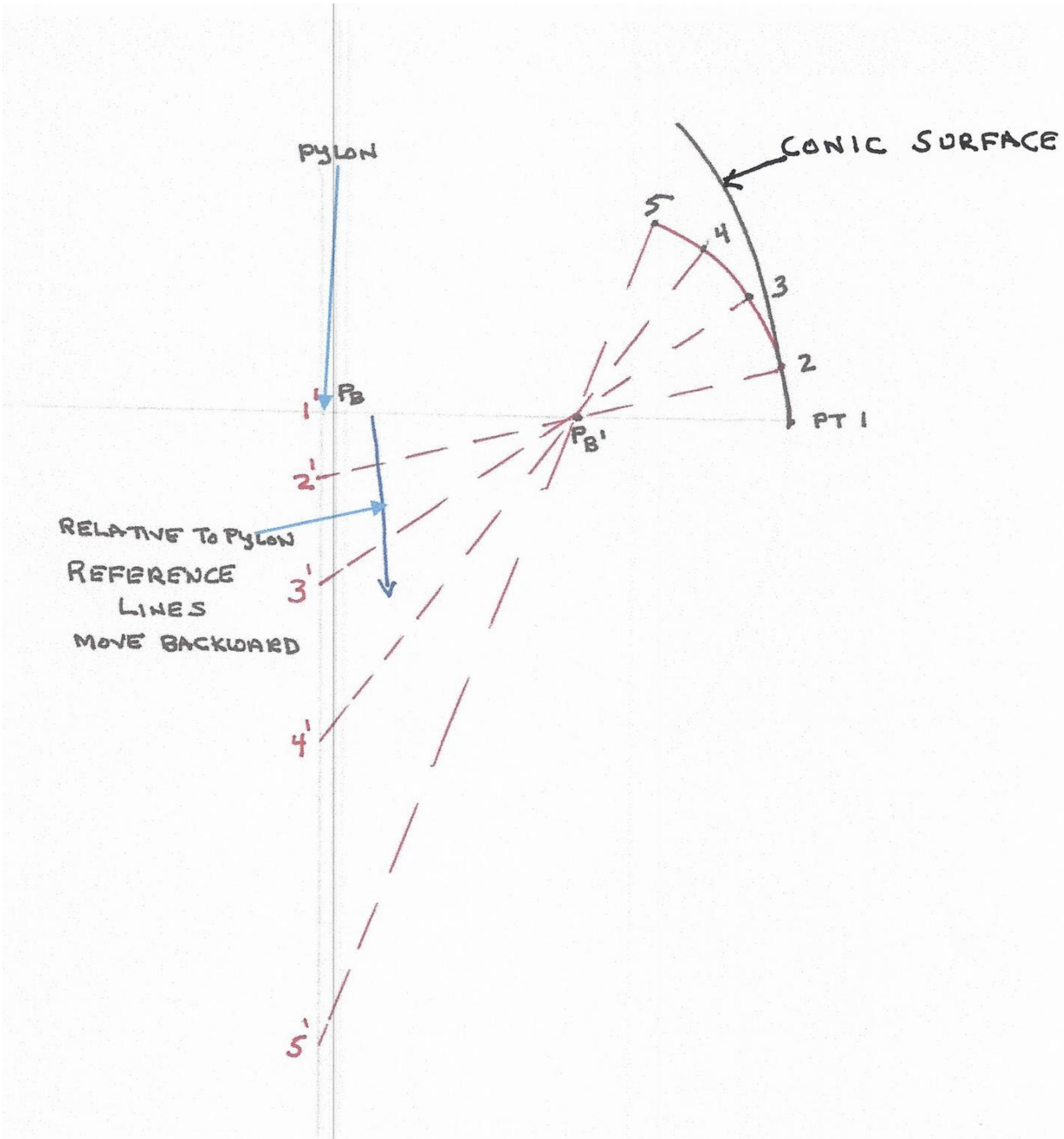

Figure 16: Reference Line Movement Relative to Pylon when Aircraft is Above the Pivotal Altitude (Looking Down on Pylon at Point P_B)

If the aircraft is below the pivotal altitude when the reference line is placed on the pylon (i.e. point 1C), the radius of the turn for the initial bank angle is larger than the radius at point 1C. Thus, the aircraft starts to turn away from the pylon. The motion of the reference line relative to the pylon is shown in Figure 17. Here, the center of the turn is now located at P'_C, and thus, as the aircraft moves from point 1 to 2, 3, and 4, the reference line is moving forward relative to the pylon (point P), i.e. points 1’, 2’,3’ and 4’. This is the indication that the aircraft is below the pivotal altitude and needs to ascend and move to a larger radius. As the aircraft climbs, the reference line will stop moving forward relative to the pylon and start to move backward until the pylon catches up to the reference line at the pivotal altitude. Again, the bank angle will need to vary in order to keep the elevation of the reference line on the pylon.

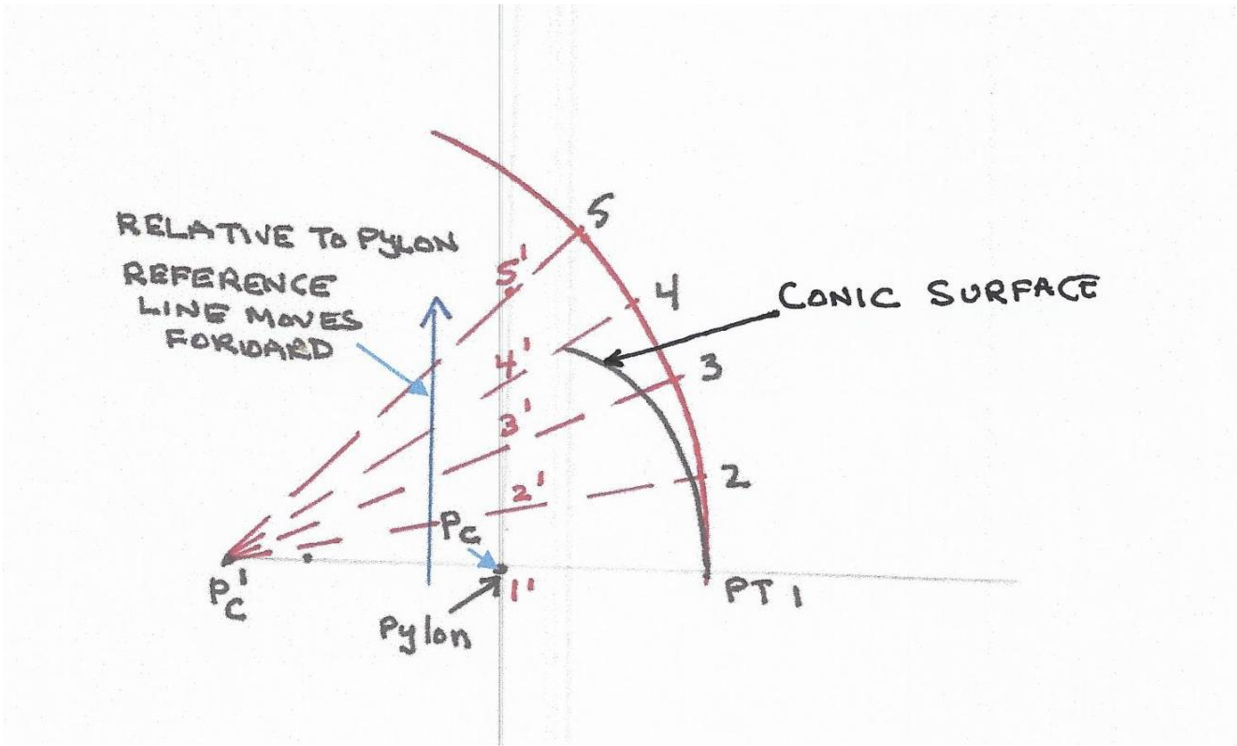

Figure 17: Reference Line Movement Relative to Pylon when Aircraft is Below the Pivotal Altitude (Looking Down on Pylon at Point P_C)

Summarizing the visual cues for determination of whether the aircraft altitude is above or below the pivotal altitude:

(1) If the reference line is moving forward relative to the pylon, or the reference line is moving in a circular motion in the direction of the turn, the aircraft is below the pivotal altitude and needs to ascend.
(2) If the reference line is moving backward relative to the pylon, or the reference line is moving in a circular motion in the direction opposite to the direction of the turn, the aircraft is above the pivotal altitude and needs to descend.

6.1.1 Planning the On-Pylon Turn with No Wind

When flying the On-Pylon Turn without a wind, it is important for the Pilot/Instructor to remember that there are an infinite number of solutions to the On-Pylon Turn maneuver, each one corresponding to a particular radius and corresponding bank angle. If the Pilot chooses a distance from the pylon to execute the maneuver, there will be a corresponding bank angle that will hold that distance. Equation (56) can be used to determine the required bank angle for a given radius. Table 5 shows the required bank angle in degrees to hold the given radius around the pylon. We also show the pivotal altitude at the bottom of the Table. Note that the numerical value of the bank angle has been rounded to the nearest degree. Here the VTAS is in knots.

Table 5: Bank Angle (Degrees)

Radius (feet)	VTAS=80	VTAS=90	VTAS=100	VTAS=110	VTAS=120
500	49	55	61	65	69
1000	30	36	42	47	52
1500	21	26	31	36	40
2000	16	20	24	28	33
2500	13	16	20	23	27
3000	11	13	16	20	23
Pivotal Altitude(feet)	566	717	885	1071	1275

Table 5 is extremely useful for the Pilot, since he/she can pick a particular radius and corresponding bank angle to set up the On-Pylon Turn. As discussed earlier, the On-Pylon Turn maneuver without a wind is also a Turn around a Point. Once the approximate radius is selected, the Pilot need only keep the elevation of the reference line on the pylon and the bank angle constant. The visual cues discussed in Section 5.1 can then be utilized to determine whether the aircraft is above or below the pivotal altitude. The Pilot should practice this maneuver with the initial altitude both above and below the pivotal altitude in order to confirm their understanding of the visual cues needed to converge on the pivotal altitude. The Pilot should also know the pivotal altitude prior to executing this maneuver. Under the no wind scenario, the pivotal altitude that the Pilot converges on for the On-Pylon Turn should be close to the value given in Table 5.

In Section 6.2 we discuss the On-Pylon Turn maneuver in the presence of a wind.

6.2 Flying the On-Pylon Turn Maneuver in the Presence of a Wind

The On-Pylon Turn maneuver in the presence of a wind is flown somewhat different than the On-Pylon Turn without a wind. In the presence of a wind, the pivotal altitude will be continually changing around the pylon. Thus, for a given windspeed ratio, the Pilot should know the pivotal altitudes for at least the four location shown in the previous Tables. These are θ =0, 90,180, and 270 degrees. Equations (33) or (34) in conjunction with Table 4 provide this information.

As shown previously, the maximum bank angle around the pylon will occur on the downwind. If the Pilot is interested in keeping the maximum bank angle below a specific value (i.e. 45 degrees), eq. (59) can be utilized to determine the radius on the downwind that will keep the maximum bank angle at this specific value.

The required maximum rate of climb in order to hold the pylon can be approximated by eq. (71). Here again, we see that the maximum required rate of climb is proportional to V_{TAS} and the tangent of the maximum bank angle on the downwind. Thus, if the given wind conditions require a higher rate of climb than the aircraft is capable of delivering, the maximum required rate of climb can be reduced by decreasing either V_{TAS} or the maximum bank angle on the downwind, or a combination of the two. We should also point out that when the V_{TAS} is reduced, the value of the windspeed ratio will increase slightly, so one needs to recalculate the windspeed ratio and then use eq. (71) to recalculate the maximum required rate of climb. It is important for all Pilots to understand that under strong wind conditions, a combination of both a reduction in V_{TAS} and bank angle on the downwind may be the only way that the aircraft's rate of climb can meet the required maximum rate of climb when holding the pylon.

We should point out that in the presence of a wind, the Pilot should enter the On-Pylon Turn on the downwind with the estimated planned distance from the pylon (i.e., controlling the maximum bank angle). The same procedures as discussed in the no-wind case should be used to hold the pylon. The altitude should be adjusted to move the line of sight of the lateral axis to the pylon and the bank angle adjusted to keep the elevation of the line of sight of the lateral axis on the pylon. The only difference between the no wind and wind case is the geometry of the conic surface will change from a circular conic to an elliptic conic.

It is also important for all Pilots to understand that there are an infinite number of solutions to the On-Pylon Turn maneuver, each having a different radius and corresponding bank angle on the downwind. Thus, if the Pilot loses the pylon and then recovers it shortly afterward, he/she may find the aircraft at a different distance from the pylon when returning back to the downwind. Note, there is no mention in Ref. 3 about maintaining the same distance from the pylon after multiple circuits. Finally, as described in Section 6, a precision On-Pylon Turn maneuver in the presence of a wind could also be flown using a constant pivotal altitude while varying the TAS.

7.0 Conclusions

In this White Paper we have derived the complete solution to the On-Pylon Turn maneuver. The solution provides the pivotal altitude, bank angle, rate of turn, rate of climb/descent, and WCA, which will hold the pylon under an arbitrary windspeed ratio, i.e. $\bar{V}_W = \frac{V_W}{V_{TAS}}$. The solution shows the pivotal altitude given in the FAA Airplane Flying Handbooks (Refs. 2 and 3) are in error when used in the presence of a wind. We show that the error is an outcome of an incorrect assumption for the Centripetal acceleration used to calculate the pivotal altitude. This error arises when the radius of the turn is not constant during the maneuver, as is the case in the On-Pylon Turn maneuver. In addition, the solution shows that the groundspeed that is utilized in determining the pivotal altitude is the transverse component of the groundspeed, i.e., it does not include the radial component of the windspeed. This is also shown to be incorrect in Refs 2 and 3.

Simple formulas are derived to allow the Pilot to setup the On-Pylon Turn so as to employ a specified maximum bank angle during the maneuver. We show that the maximum bank angle occurs on the downwind when the aircraft is closest to the pylon, while the minimum bank angle occurs on the upwind when the aircraft is farthest from the pylon. The percent change in pivotal altitude from the downwind to the upwind is shown to be equal to $0.2\bar{V}_W$, and thus is proportional to the windspeed ratio. At high windspeed ratios, i.e. ≥ 0.2, with the aircraft flown at 90 KTAS, the required maximum rate of climb of the aircraft to hold the pylon can exceed the aircraft performance capability of a C-172. We have derived a formula for the required rate of climb/descent to hold the pylon. This formula allows the Pilot to determine the necessary reduction in V_{TAS} or bank angle on the downwind, in order to reduce the required maximum rate of climb such that the required aircraft performance is consistent with the aircraft capabilities. In addition, we have also shown that the On-Pylon Turn in the presence of a wind can be flown at constant pivotal altitude by varying the TAS around the pylon. If the On-Pylon Turn is entered on the downwind, the only difference between the constant TAS and varying TAS methods is that in the case of the varying TAS method, the aircraft remains closer to the pylon in the region θ =90 to 270 degrees, and exhibits larger turn rates and bank angles around the pylon. However, the required bank angles around the pylon are always less than that required on the downwind.

We have also derived the physical explanation for the visual cues that the Pilot observes when the aircraft is above or below the pivotal altitude, and the necessary corrections to move the reference line back onto the pylon. Finally, we have provided some helpful information on setting up and flying a precision On-Pylon Turn maneuver.

8.0 References

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(2) FAA Airplane Flying Handbook (8083-3A), 2004

(3) FAA Airplane Flying Handbook (8083-3B), 2016

(4) Thomson, W.T., "Introduction to Space Dynamics", Dover Publication, 1961.